International Trade

and the

National Income Multiplier

by

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PREFACE

The Aims of this Book are threefold. One of the objectives is to reconsider those parts of the theories of international trade and capital movements which can be profitably analyzed with the technique of the money-income multiplier. Furthermore, an attempt is made to develop the multiplier technique in certain respects, applicable not merely to the theory of foreign trade but also to that of the trade cycle and to monetary problems in general. Lastly, the book has the pedagogic objective to present the multiplier pattern in so patient an exposition that it loses all the horrors which it still seems to hold for many, and becomes an easy tool which the student understands to handle with facility and care.

The book therefore is offered both as a teaching medium and as a technical contribution.

The Problems for Discussion are primarily those of the effects upon national income and trade balances which may result from (1) "autonomous" changes in exports or imports, (2) changes in exports induced by changes in incomes abroad, (3) inverse changes in investments at home and abroad, especially those produced by autonomous capital movements, and (4) parallel changes in investments at home and abroad. As intermediate products or as by-products of these analyses, several other matters are included in the discussion. For example, the general theory of international capital movements must be reexamined in order to gain a fuller understanding of the interrelationships among foreign lending, home investment, foreign trade and national income. Valuable clues are obtained for the analysis of the transfer problem. All processes are closely tied up with the working of banking mechanisms, first by way of a preparatory exposition, then by

way of observations with respect to particular situations.   The
attempts of computing the money-income multiplier from
statistical data are critically examined.   The assumptions of
stable commodity prices, wage rates and interest rates are
analyzed as to their methodological usefulness and to the
limitations of the probability value of the results obtained.
Finally, the "protectionistic" and "imperialistic" interpreta-
tions of the theory of the multiplier are appraised.

A prefatory "justification" for my undertaking is, I believe,
not necessary.   But I cannot help quoting, in substitution for
any justification that I might have formulated, a remark which
a recent writer in the field made at the end of his study.   "In
conclusion," he said, "the opinion may be hazarded that
perhaps the time has come to rewrite the theory of inter-
national trade in terms of the national money income."[1]   I
have not rewritten the theory of international trade, but I
have discussed some phases of it "in terms of the national
money income."

The Multiplier Technique is at the moment the most efficient
way of dealing with effects of changes in disbursements, such
as additional investments or exports, upon national money
income.   To be sure, this multiplier technique is greatly
distrusted by many.   I use it myself only heavily protected
with an armor of qualifications and reservations.   I would not
even dare to contradict that skeptical economist who expressed
the belief that "it will not be long . . . before the present
fashion for multipliers, relations and independent variables
passes."[2]   This may possibly be the case, provided that

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[1] Charles Poor Kindleberger, *International Short-Term Capital Movements*,
New York, 1937, p. 237.

[2] Professor L. G. Melville in an address before the Economic Section of the
Australian Association for the Advancement of Science, January 1939,
quoted by Mr. Colin Clark, "Comment," *Economic Journal*, Vol. 49, 1939,
p. 357.   Professor Melville certainly went too far when he included inde-
pendent variables among the things subject to whimsical and ephemeral
fashions.

techniques superior to those favored by the present "fashion" are developed. But until then we had better use what we have.

I have tried to improve the multiplier technique by discarding the idea of the instantaneous or timeless multiplier and introducing time as an important variable. I believe that period analysis, showing the step-by-step adjustments of incomes, imports and exports as sequences in time, is best adapted to the present purposes. And, incidentally, I submit that the study of these movements from period to period constitutes really "dynamic economics."

The Model Sequences and Tables in which the step-by-step adjustments are pictured make a careful reading of this book rather laborious. Eleven different models are presented, most of them with several variations. Thus, no less than thirty tables are shown.

Those who will not take the time to study the tables in detail may find them rather tiresome. But those who plod through them conscientiously are likely to be increasingly interested or even—as some students on whom I tried them out—fascinated about certain developments—equalities, proportions and dimensions—shown in the sequences of figures. In my experience with students, post-graduate as well as undergraduate, I found that these model sequences with their seemingly "dull" arithmetic are not merely instructive but well-nigh indispensable for a thorough understanding of the processes involved in multiplier theory. Even students who had gone through the writings of several expounders of the multiplier principle, did not really grasp it fully until they trudged through model sequences of the sort presented in this book.

The Algebra and the Formulas could not be avoided in an analysis of this sort. The equations were not used in order to make the study look more learned or elegant, but they were

needed for arriving at results which by literary reasoning could be reached, if at all, only at the expense of vastly more time and space.

With few exceptions, the "math" employed in this book is on the level of intermediate high-school algebra. This is true even for the mathematical appendices. And, to the great annoyance of mathematical economists, I have not made any jumps or big steps which the tenth-grader could not follow. Almost every single substitution, transformation, rearrangement is explained, and no higher mathematics is involved.

The Development of this Book, starting from a few observations which I made on the so-called leakage through imports in an article in the *Quarterly Journal of Economics* in 1939,[1] has mostly taken place in class-room discussions at the University of Buffalo. Early results were presented in papers to the Economics Club of Yale University in March 1941 and to the International Relations Seminar at Harvard University in October 1941. It was on the latter occasion that my attention was drawn to an excellent essay by William A. Salant on "Foreign Trade Policy in the Business Cycle."[2] In spite of the surprising similarities in the two studies, they had developed quite independently of each other.

Some parts of my analysis—those dealing with capital movements and transfers—have their roots in earlier articles, which I published in various places in 1928, 1930 and 1932. But I am not ashamed to admit that I have revised my views in several respects.

Acknowledgments must first be made to those whose work has had the strongest influence in the gradual revision of my thoughts on this subject. Lord Keynes has forced almost

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[1] "Period analysis and multiplier theory," *Quarterly Journal of Economics,* Vol. 54, November 1939, pp. 1–27. The article is reprinted in *Selected Readings in the Theory of Economic Fluctuations,* Philadelphia, 1943.

[2] *Public Policy,* Vol. II, published by the Graduate School of Public Administration, Harvard University, 1941.

everybody in our field of study to reconsider earlier theories. One does not have to be a "Keynesian" to admit the influence of Keynes. Indeed, one may maintain a critical attitude concerning many Keynesian ideas and yet may acknowledge one's indebtedness to them for the important catalytic functions which they have performed.—Much footnote space in this book is devoted to the statistical studies of Mr. Colin Clark. That my remarks are mostly critical in tone will, I hope, not be taken as an indication of disrespect for Mr. Clark's courageous attempts in econometrics.

Parts of the manuscript of this book—ranging from single sections to several chapters—were read by friends of mine. Most of them offered valuable criticism, others checked my algebra, some advised me in the arrangement of the tables. For these aids I am indebted to M. A. Brumbaugh, P. T. Ellsworth, Milton Friedman, Harry M. Gehman, Gottfried Haberler, Folke Hilgerdt, Agnes Roman, Paul Samuelson, Arthur Smithies, Harold M. Somers and Joseph Ullman. For competently editing the whole book and for valuable suggestions in matters of exposition I wish to thank my friend Dr. Horace G. White, Jr. The responsibility for all shortcomings of the book is, of course, exclusively my own.

<div style="text-align: right">FRITZ MACHLUP.</div>

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PREREQUISITES

THE CIRCULAR FLOW OF MONEY

The notion of a "circular flow of money" in the economic system is misleading in some respects, helpful in others. It may mislead if one overlooks the fact that the "flow" is governed by the decisions, habits and preferences of the individuals and firms who receive, hold and disburse money. It may mislead also if one overlooks that there is not one circuit but rather a large number of longer and shorter circuits interfused in a bewildering complex. But some significant aspects of the maintenance, growth or decline of the national money income can be fairly adequately described even by the simpler forms of the circular-flow concept.

The circular flow maintains its size if the money is always "passed on" in regular intervals. More correctly, the flow of money passing from hand to hand and account to account remains at a given level if all the money that is received (in period t_1) by individuals or firms is again disbursed by them (in period t_2); or, if for all the money that is received (in t_1) but not redisbursed (in t_2) by the recipients an equal amount is disbursed (in t_2) by others who have not received it (in t_1). We need not enter here into the question of determining the significant "periods" or "intervals"—which for many problems is crucial, because the money flow is pulsating rather than continuous. But, to make clear the nature of the money flow, we should discuss here the types of money receipts and money disbursements.

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Money is received (a) by income recipients, (b) by disinvestors, (c) by borrowers, debt collectors and sellers of securities, and (d) by collecting agents.   Money is disbursed (a) by consumers, (b) by investors, (c) by lenders, debt payers and buyers of securities, and (d) by collecting agents.[1]

Money received by an income recipient may be disbursed by him as a consumer, investor, lender, debt payer, or securities buyer.   Money spent by a consumer is received in rare cases by another income recipient (e.g. a domestic servant) but in most cases by a disinvestor; for it is a disinvestment if a storekeeper reduces the stock of goods on his shelves, even if he re-invests quickly afterwards by replenishing his inventory. The largest part of the sales proceeds of firms are disinvestments; and, in turn, the largest part of all disbursements by firms are investments.   (In a stationary state the disinvestments and reinvestments are equal, leaving net investment zero.)   The funds disbursed by firms as investors are received partly by individuals as income recipients (for services rendered) and partly by other firms as disinvestors (for goods sold).   Doubts may arise as to the proper classification of net profits, that is of the part of the sales proceeds of firms which is in excess of cost and, hence, of their disinvestment.   With regard to those portions of the net profits which a firm is going to distribute as dividends, it may most conveniently be considered as a collecting agency for the shareholders;[2] and only

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[1] From this classification it is clear that so-called "financial investment" is grouped with lending, and that only "real investment" belongs under (b).   But it should be understood that real investment includes investment in inventory and goods in process (e.g., through payment of wages).

[2] Net profits to be distributed, parts of the sales proceeds of the firm, are not disinvestment, for they are in excess of the firm's investment in the production of the goods sold; they are not net income either, for this would be double counting since the dividend payments are regarded as income of the shareholders.   Hence the device to view the firm as collecting agency, collecting the net profit (from the purchaser of the goods sold) for distribution to the shareholders.

with regard to undistributed profits should a firm be considered as an income recipient.[1]

If the picture of the circular flow is confined to the national economy, receipts from exports and disbursements for imports must be considered as injections into the flow and ejections from it, respectively. The money injections through exports are, in the first place, receipts by exporters as disinvestors (selling stocks of goods abroad); but indirectly, through the investment by the firms which produce the export articles, the money will be received by income recipients. (We must bear in mind that this investment is not in fixed capital equipment but merely in goods in process and inventories.) The money ejections through imports are, in the first place, disbursements by importers as investors (buying stocks of goods from abroad); but indirectly, if consumers' goods are imported, the money comes from consumers' disbursements. As long as exports and imports remain equal, no net change in the money flow results from these injections and ejections.

In order to arrive at shorter and simpler statements, we shall for the moment leave out the lenders and borrowers, debt payers and collectors, securities buyers and sellers, and the collecting agents. With all these people out of the way, two simple statements become obvious:

(1) If exports and imports are equal, the sum of disbursements by consumers and investors (in period t) is identical with the sum of receipts by income recipients and disinvestors of the same period (t).

This should be easy to understand. What somebody disburses somebody else must receive (especially in the case of account-to-account money, which cannot be lost in transit). With consumers and investors as the only disbursers, and

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[1] Precisely this is the procedure followed by national income statistics. See, for example, Simon S. Kuznets, *National Income and Its Composition, 1919–1938*, New York, 1941.

income recipients and disinvestors as the only recipients of money, the identity between the disbursements of the former and the receipts of the latter is self-evident.   And, if the excess of investors' disbursements and disinvestors' receipts is called net investment, it follows that income receipts must be identical with the sum of consumption expenditures and net investment.[1]

The second of the announced statements refers to the maintenance or change of the money flow and must therefore include a comparison of two periods: for, we repeat, the money flow is maintained if all that is received in one period is disbursed in the next.   Thus we can state:

(2) Even if exports and imports are equal, the sum of disbursements by consumers and investors (in period t) is not necessarily equal to the sum of receipts by income recipients and disinvestors in the preceding period $(t - 1)$; only if these sums are equal will the money flow remain constant; otherwise it must rise or fall.

There can be no more difficulty about this statement than about the first one.   And when the two statements are put together—identity of current receipts with current disbursements, and possible equality or inequality of current disbursements with previous receipts—the dependence of the money flow upon the comparative sizes of current consumption and investment disbursements and previous income and disinvestment receipts is more than obvious.[2]

We must now expand the two statements to take account of possible differences between exports and imports.   If some of

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[1] If one now chooses to define "saving" as the excess of the income receipts over consumption expenditures of the same period, it follows logically that "saving" is identical with net investment.

[2] If one now chooses, in contradistinction to what was done in the preceding footnote, to define "saving" as the excess of the income receipts of the preceding period over the consumption disbursements of the current period, and to define "net investment" as the excess of the investors' disbursements

the disbursements do not go to domestic recipients but "go abroad" to pay for imports, and if some of the receipts do not come from domestic disbursers but "come from abroad" in payment for exports, we must say, in place of (1):

(3) The sum of disbursements by consumers and investors plus the proceeds from exports minus the payments for imports (in period t) is identical with the sum of receipts by income recipients and disinvestors in the same period (t).

If the excess of proceeds from exports over payments for imports is called export surplus and if the excess of investors' disbursements over disinvestors' receipts is called net investment, it follows that income receipts must be identical with the sum of consumption expenditures, net investment and the export surplus.[1]

The second statement needs an analogous expansion in order to take account of foreign-trade balances. Concerned with the constancy or change of aggregate money receipts we must compare what is received in one period with what was received in the preceding period. Currently received, according to statement (3) is what consumers and investors disburse plus what is obtained from exports minus what is paid for imports. Comparing these current receipts with what consumers and disinvestors "took in" in the preceding period, the statement replacing (2) must read:

(4) The sum of disbursements by consumers and investors plus the proceeds from exports minus the payments for imports (in period t) is not necessarily equal to the sum of receipts by income recipients and disinvestors in the preceding

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of the current period over the disinvestors' receipts of the preceding period, it follows logically that the money flow remains constant if "saving" is equal to "net investment," whereas it will rise with an excess of "net investment" and fall with an excess of "saving."

[1] If "saving" is defined, as in footnote 1 on page 4, as the excess of the income receipts over consumption expenditures of the same period, it follows logically that "saving" is identical with the sum of net investment and export surplus.

period $(t - 1)$; only if they are equal will the money flow remain constant; otherwise it must rise or fall.[1]

A RISING FLOW OF MONEY INCOME

It was perhaps an unnecessary caution when we spoke merely of the money flow instead of the income flow even after we had excluded, by a simplifying assumption, the possible influences through changing volumes of transactions among lenders and borrowers, debt collectors and payers, securities sellers and buyers, and collecting agencies and their principals. With an auxiliary assumption concerning a given degree of financial integration (which controls the ratio between disinvestors' receipts and net income receipts, or the average number of firms through which money spent by income recipients has to pass before it becomes again income) the translation of "flow of money" into "flow of money income" becomes quite safe—if it is not anyhow sufficiently protected by the flexible definition of the "income period" to which the receiving and passing on of income refers.

The four statements above have made it clear that increased consumption disbursements, increased net investments and improved trade balances will cause the flow of money income to rise. But we must not be satisfied with this simple truth.[2] We can know more than that an increase in net investments, with unchanged consumption and trade balance, will result in higher money incomes. For example, we may learn that

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[1] If "saving" is defined, as in footnote 2 on page 4, as the excess of the income receipts of the preceding period over the consumption expenditures in the current period, and net investment as the excess of investors' disbursements of this period over disinvestors' receipts in the preceding period, it follows logically that the money flow will be constant, rising or falling according to whether "saving" is equal to or smaller or greater than the sum of "net investment" and the export surplus.

[2] It is a "truism," if all necessary assumptions are stated and all necessary qualifications made; it is a "probability," if the list of assumptions and qualifications is incomplete.

consumption and trade balance will not remain unchanged if net investments increase; and we can perhaps find out by how much they are likely to change with a given increase in net investments. And knowing, or assuming that we know, the relationships between these changes, we may arrive at the probable ratio between the resultant income increase and the original increase in net investment.

For the analysis of these relationships the technique of the income multiplier was developed.[1] This technique does not deal with the effects which a single dose of additional disbursements would have upon the national income; indeed, the theory contends that a merely temporary increase in disbursements would have no permanent effect upon incomes, for what is once injected into the income flow would be lost again through certain "leakages." Only a lasting increase of primary disbursements (let us say, additional investments or additional exports, repeated in certain amounts period after period) can cause the income flow to rise to a higher level and to maintain it thereafter.

A new, continuous or periodic trickle enters a circular flow; to think that the flow should rise by exactly the volume of one dripping would be clearly wrong; but to think that the circular flow should rise steadily from then on and keep rising as long as the trickle lasts, would not be correct either. What really happens is this: The trickle adds to the flow, but more leaks develop as the flow rises. As the circular flow reaches a certain level, the leakages will just offset the trickle, and the continued trickle will just maintain the flow at its higher level. What is then the ratio between the increase in the flow and the

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[1] The first exposition of multiplier theory giving rise to an uninterrupted discussion in recent years was in an article by Mr. R. F. Kahn on "The relation of home investment to unemployment," *Economic Journal*, Vol. 41, 1931, pp. 173–198. The theory gained widest currency in the form it was given by Lord Keynes in his *General Theory of Employment, Interest and Money*, London, 1936, pp. 115 ff.

trickle? By what figure may one multiply the volume of the trickle to obtain the volume which is added to the circular flow?

This metaphoric language conveys, I believe, the basic ideas which underlie the theory of the multiplier. The "trickle" is, of course, nothing but the primary addition to income through expansionary consumption or investment or through an improvement of the foreign-trade balance. The trickle is continuous or periodic, just as the said primary contributions to income are assumed to be continuous or periodic. The leakages which develop as the income flow rises are chiefly (but not solely) the induced increases in saving and in imports which can be expected on the basis of people's marginal propensities to save and to import. As their incomes rise, people save more and import more; hence, not all that they receive is passed on in the domestic income flow. Eventually, the income flow reaches a level at which the leakages just offset the periodic injections, so that the continued injections will just maintain the income flow at its higher level. And we ask ourselves what is the ratio between the increment in income (per period) and the additional disbursement or export (per period). In other words we ask what is the income multiplier, the number by which we may multiply the amount of the periodic injection into the income flow if we wish to know the amount by which the income flow is expected to rise.

REPERCUSSIONS, MULTIPLIER AND MULTIPLICAND

Let us reiterate in more formal language: the theory of the income multiplier states that, and explains why, additional disbursements of a certain type, starting now and repeated henceforth in equal periodic amounts, will generate additional incomes per period in amounts which (ultimately) will be a

multiple, and indeed a determinate (predictable) multiple, of the said disbursements.

Some of the major criticisms raised against this theory concern, first, its assumption of given and constant "marginal propensities" which determine the value of the multiplier, second, the neglect of the time needed for adjustment of the income flow to the changed rate of disbursements and, third, the neglect of a number of repercussions which the additional disbursements are liable to bring forth in the economy.[1] These and other objections may indeed discredit the theory in so far as its application to concrete situations is concerned, but its value for demonstrating possible relationships remains unimpaired. That is to say, the explanatory and pedagogic value of the theory may be considerable even if its predictive value is small. Moreover, if multiplier theory is sufficiently qualified and reformulated to take account of varying propensities, inherent time lags and probable repercussions, its predictive usefulness may be much enhanced.

One set of necessary qualifications concerns the probable repercussions which depend on the specific type of the income-creating disbursements. In early formulations government deficits, private investments and improved trade balances were all lumped together under the heading of "investment":[2] but this will not do as soon as it is recognized that the repercussions are likely to be different in the three cases. The "government-expenditures multiplier," the "private-investment multiplier" and the "foreign-trade multiplier" may be seen to have different values for one and the same economy

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[1] See, among others, Gottfried von Haberler, *Prosperity and Depression*, third revised edition, Geneva and New York, 1941; Henry H. Villard, *Deficit Spending and National Income*, New York, 1941; and my own article "Period analysis and multiplier theory," *Quarterly Journal of Economics*, Vol. 54, 1939–40.

[2] Government deficits were appointed "honorary investment," as Professor D. H. Robertson once remarked. "Mr. Clark and the foreign trade multiplier," *Economic Journal*, Vol. 49, 1939, p. 354n.

at one and the same time, if dependent changes and repercussions are no longer assumed away or surreptitiously hidden away by *ex post* corrections of the multiplicand.[1]

In this study, which concentrates attention on the foreign-trade multiplier and on the foreign-trade aspects of the home-investment multiplier, one type of repercussion will be carefully dealt with: the repercussion upon exports which is induced through dependent changes of incomes abroad. For the sake of simplicity, the assumption of stable marginal propensities will be maintained throughout the study (see explanation on page 20). Time coefficients, however, will be given careful consideration: not a timeless analysis but rather a sequence analysis (period analysis) will be employed. And all "secondary" ("induced," "consequential," "equilibrating") phenomena will be separated from "primary" ("original," "autonomous," "spontaneous," "disturbing") ones.

We shall employ the good old method of gradually diminishing abstraction, that is to say, we shall proceed from very simple models to more complicated ones. The first model, for instance, abstracts from induced savings and foreign repercussions; the second and third models abstract from only one of these two crucial determinants of the multiplier; the fourth model includes them both, but, as all the previous models, abstracts from the existence of more than two countries; the

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[1] All of the statistical "determinations of the multiplier" in earlier writings were in fact *ex post* determinations of the multiplicand, because most of the repercussions which a certain amount of autonomous income-creating disbursement may have had were added to or deducted from that amount. Thus, instead of finding the multiplier as the ratio of "net effect" to "autonomous cause," one part of the "effects" was shoved back into the "cause," thereby, with hindsight, inflating or deflating the multiplicand. Consequently the resulting multipliers are of little use in answering questions like these: "By how much will the national income rise if public works costing so-and-so many billion dollars are undertaken?" or, "By how much will income rise if, through export subsidies, additional exports in a value of that-and-that many million dollars are obtained?"

fifth model includes several countries but adopts other simplifying assumptions, which are relaxed in the sixth, and finally dropped in the seventh model. The succeeding models deal with cases of a somewhat different type. While the first seven models relate only to so-called autonomous changes in foreign trade, the eighth, ninth and tenth models deal with cases where changes in foreign trade are connected with changes in home investment and international capital movements.

This slow approach may be considered overly cautious by some skilled mathematical economists who are well trained in juggling with a large number of variables. However, I believe, most students of economics will feel safer and more comfortable if we proceed slowly and, occasionally, take time out to look around and reflect.

AUTONOMOUS AND INDUCED FOREIGN TRADE

There are interesting relationships between foreign trade and the home-investment multiplier; for example, import appears as one of the leakages from the income flow resulting from home investment. The propensity to purchase foreign goods with some portion of an increment of income, along with the propensity to save a portion of that increment of income, causes the marginal propensity to consume home-made products to be so much smaller than unity. These main leakages, the one through saving, the other through imports, cause the series of re-spendings starting from each dose of new income gradually to dwindle, and this dwindling prevents the secondary income, generated from a primary increase in "investment," from growing indefinitely.

The foreign-trade leakage from the secondary income flow should not be confused with an "autonomous" change in the foreign-trade balance, which is in its own right an income-creating disbursement. That is to say, foreign trade as a

leakage from secondary incomes must be distinguished from foreign trade as a generator of primary income. It is the latter which plays the leading part—the title role—in the theory of the foreign-trade multiplier. In other words (putting it so that both directions of change are covered by our formulation) an autonomous increase or decrease of exports or decrease or increase of imports (as generator or destroyer of *primary* income), constitutes the *multiplicand* on which the foreign-trade multiplier operates; the *ultimate* net change of income comes out as the *product* of that multiplication.

An increase or decrease in exports or imports is here called autonomous if it is not induced by a change in income.[1] Changes in consumers' preferences or in producers' conditions of production, changes in transport costs, tariffs or other trade barriers, may all result in "autonomous" changes in exports or imports. "Induced" changes in exports or imports then are those which can be attributed to changes in income and, thus, are explained by the income-elasticity of the demand for foreign goods.[2]

A complete analysis would not confine itself to income-induced changes but would also include price-induced changes in exports and imports if the price changes in question were in turn the result of "autonomous" changes in exports and imports. It has become fashionable to neglect these potentially resulting price changes and their consequences; the assumptions of unemployed resources, inflexible wage rates and perfectly elastic supply curves (and, it must be added, stable foreign-exchange rates), are used as excuses for "assuming away" any repercussions via price. Whether these assumptions are warranted or not, there is certainly good

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[1] *Cf.* Colin Clark and J. G. Crawford, *The National Income of Australia*, London, 1939, p. 95.

[2] Just as the changes which we call autonomous may have their roots at home or abroad, the induced changes in trade may be due to income changes at home or abroad.

sense in isolating income effects from price effects. The "propensities" on which multiplier theory is based are all defined in terms of income changes. In conformity with this practice, only the directly income-induced adjustments in exports and imports are treated here as induced exports and imports.[1]

The "leakage" through induced imports enters, of course, into the determination of the foreign-trade multiplier just as into that of any other income multiplie . No matter whether the primary increase in income was due to home investment or to an autonomous increase in exports, the leakage through increased imports will help in restricting—and finally arresting—the growth of the secondary income flow which has sprung from the additional disbursements.

Foreign trade plays, thus, a double role in foreign-trade multiplier theory: once as multiplicand, and secondly as one of the determinants of the multiplier. This double role of foreign trade, I believe, is likely to defeat every attempt at statistical verification of foreign-trade multiplier theory. For, unfortunately, the theoretically indispensable separation of the autonomous changes in foreign trade—constituting the multiplicand—from the induced changes in foreign trade—affecting the multiplier—would seem to be in most cases statistically impracticable.[2] More about this will be said later.

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[1] For a similar defense of this method see Lloyd A. Metzler, "The transfer problem reconsidered," *Journal of Political Economy*, Vol. 50, 1942, p. 398. The question of the neglect of price effects will be taken up below in several places; see especially Chapter X.

[2] Mr. Colin Clark thought otherwise. In the study on *The National Income of Australia, op. cit.*, by Colin Clark and J. G. Crawford, all changes in exports were considered as "autonomous," while changes in imports were for some years divided into "autonomous" and "consequential" ones and for other years counted as entirely "consequential." In his study on Great Britain, however, Mr. Clark conceded that no satisfactory separation appeared to be possible. See his article, "Determination of the multiplier from national income statistics," *Economic Journal*, Vol. 48, 1938, p. 439, and his book, *The Conditions of Economic Progress*, London, 1941.

SOURCES OF THE MONEY FLOW, LEAKAGES AND SEEPAGES

The student of money and banking should never be satisfied with statements about income-creating disbursements and income-restricting leakages, unless he can visualize the monetary mechanisms which are involved in these phenomena. In other words, a translation must be possible into terms of credit creation (or avoided credit cancellation) and dishoarding, on the one hand, and credit cancellation (or avoided credit creation) and hoarding, on the other hand.[1]

In order to be net income-creating a disbursement must be met out of newly-created or newly-activated funds; this will usually mean that either banks or owners of idle funds acquire assets. In the case of home investment these assets ordinarily are loans or securities.[2] In the case of a foreign balance (through increased exports or decreased imports of goods or services) the assets acquired by the banking system or by owners of idle funds will ordinarily be foreign loans (or loans to finance foreign lending), balances in foreign banks, gold, or perhaps hitherto foreign-held securities.[3] The case in which

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[1] I emphasize this double-check method of analysis in order to avoid the dangers of one-sided thinking. There are those who think only in terms of money-supply and who are apt to jump to conclusions which would require a study of investment decisions and consumption habits. And, on the other side, there are those who think only in terms of investment incentives and consumption propensities and are apt to jump to conclusions which would require a study of money and banking mechanisms.

[2] In this case, if banks are financing the investments, the accounts of the banking system would show one or both of the following changes:

All banks		All banks		
Loans	+	Deposits	+	
Securities	+	Deposits	+	

[3] The accounts of the banking system would show any of the following

foreigners have been holding balances in the exporting country and are reducing these balances in payment for their purchases, can be regarded as a case of either creation or activation of money, depending on whether bank deposits of foreigners are or are not regarded as a part of the quantity of money.[1]

In order for a disbursement to be net income-reducing it must result in the cancellation or inactivation of funds; this will usually mean either that the banks sell assets and thereby reduce customers' deposits or that other owners of assets sell them and hold the proceeds idle. In the case of a negative

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changes:

(1)	All banks			(2)	All banks		
Loans	+	Deposits	+	Due from foreign banks +		Deposits	+

(3)	All banks			(4)	All banks		
Gold	+	Deposits	+	Securities	+	Deposits	+

Of course, the mechanisms observed in reality may look more complicated at first sight. For instance, the export from country A to country B may be financed by country C, in that the exporter's draft on the importer is discounted by a bank in C. (See, among others, Marco Fanno, *Normal and Abnormal International Capital Transfers*, Minnesota, 1939, p. 95.) We have then no less than three international credit transactions before us; a loan of A to B (deferring the payment for the goods), a loan of C to A (discounting the draft) and a loan of A to C (accepting a deposit in the bank of C). It is the third of the three transactions which creates funds in A when the exporter sells his foreign bank balance to his domestic bank. Of the four T-accounts which are shown in this footnote, the second one would reflect this case.

[1] The accounts of the banking system, in this case, would show the following changes:

All banks		
	Deposits	+
	Due to foreigners	

foreign balance (through decreased exports or increased imports of goods and services) the assets disposed of by the banking system or by accumulators of idle funds will ordinarily be foreign loans (or loans to finance foreign lending), balances in foreign banks, gold, or perhaps securities which are sold to foreign buyers. The case where foreigners are building up balances in the importing country may be treated, again, either as a cancellation or as an inactivation of money, according as to whether bank balances of foreigners are or are not regarded as part of the circulation.[1]

Leakages from the secondary income flow are not "net income-reducing," but they merely offset simultaneous disbursements which otherwise would create additions to the new money. Induced savings need not take the form of hoarding or of a cancellation of bank credits; the savers may acquire the assets which would otherwise be acquired by the banking system or by owners of idle funds. Thus, induced saving would offset primary investment, or an autonomous export increase or import reduction, either by cancelling or inactivating the money funds which are created or activated through the primary disbursements, or by direct financing of the primary outlays.

Likewise, induced imports would operate as offsets to primary disbursements. For example, if home investment is financed through the acquisition of government bonds by the banks, and payment for induced imports is made out of foreign bank balances held by domestic banks, the result, in part, will

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[1] The accounts of the banking system, in this case, would show the following changes:

All banks	
Deposits	—
Due to foreigners	+

be a switch in the composition of bank assets.[1]   Or, if home
investment is financed through bank loan expansion, and if
induced imports lead to increased balances of foreigners in the
banks of the importing country, the leakage will consist in a
growth of foreign-held deposits at the expense of active
domestic deposits.[2]

There is, of course, also the opposite phenomenon, the
counterpart of the leakage, for which the term seepage may be
suggested.   The leakage, a decline in successive spending
from an enlarged money flow, prevents income from growing
continuously toward infinity.   The seepage, a decline in
successive non-spending from a reduced money flow, prevents
income from shrinking continuously toward zero.   Just as the
leakage from the income flow offsets primary income-creating
actions and ultimately arrests the income growth, the seepage
into the income flow offsets primary income-reducing actions
and ultimately arrests the income decline.

This is how the seepage works.   If, for example, an autono-
mous fall in exports reduces primary income, and if the failure
to receive income spreads from person to person, the disap-

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[1] The accounts of the banking system would show the following changes:

All banks

Securities	+	Deposits	+/−
Due from foreign banks	−		

The amount of the net change in total deposits depends on the absolute
changes in the other accounts considered.

[2] The accounts of the banking system would show the following changes:

All banks

Loans	+	Deposits	+/−
		Due to foreigners	+

The amount of the net change in total deposits depends on the absolute
changes in the other accounts considered.

pointed recipients of reduced incomes, as a rule, will not cut down their purchases of domestic products by the full amount by which their incomes have fallen.   They will probably save less than before (they will, in some income groups, even dissave) and their demand for foreign products will also be reduced.   Thus, the marginal propensity to save and the marginal propensity to import operate here in reverse, the income decline resulting in an induced decline in saving and an induced decline in imports.   If the fall in exports reduces the assets of the banking system, e.g., through an outflow of gold, the reduction of the volume of deposits may be gradually arrested not only by the induced decline in imports but also by the induced decline in saving.   This decline in saving may involve dishoarding by dissavers, but, more likely, it will mean that the banking system will have the opportunity of making loans and investments which otherwise would have been made by the savers directly,[1] or that an expansion of the earning assets of the banking system will not be offset by an otherwise occurring inactivation of savers' bank balances.

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[1] In this case, the accounts of the banking system would show the following changes:

All banks

| Reserves (Gold) | − | Deposits | −/+ |
| Loans, securities | + | | |

The amount of the net change in total deposits depends on the absolute changes in the other accounts considered.

Chapter II

ASSUMPTIONS. THE FIRST MODEL: INDUCED IMPORTS. A DIGRESSION ON CLASSICAL THEORY

ASSUMPTIONS FOR A DEMONSTRATION OF THE MODUS OPERANDI

The foregoing discussion of the monetary aspects of income-creating and income-reducing changes in investment and foreign trade, and of the leakages from and seepages into the income flow, should serve as an aid in understanding the mechanisms involved in the *modus operandi* of the foreign-trade multiplier.

In the demonstration of the *modus operandi* we shall make use of a number of simplifying assumptions, in order to isolate particular phases of the problem. In all the model sequences which will be worked out in this study the assumptions will be made (1) that the assumed propensities to save and to import are absolutely stable throughout the periods under consideration; (2) that prices (including foreign-exchange rates and interest rates) are also stable; and (3) that home investment is entirely unaffected by the changes in foreign trade as well as by the changes in income which may result from foreign trade. Hence, interest rates as well as investment incentives remain unchanged, home investment per period is constant (i.e., its changes are zero) and the rate of saving depends on nothing but income. By means of these assumptions the income effects of autonomous changes in foreign trade are isolated from all effects of changes in home

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investment and changes in psychological propensities. (Of course, effects of changes in income distribution are also ruled out.)

CONCERNING ASSUMPTION (1):

A good discussion of whether or not the assumption of stable marginal propensities to *save* is legitimate can be found in Professor Haberler's *Prosperity and Depression*, 3rd Edition, pp. 228–31. To assume that the marginal propensities to *import* are given and stable is an equally doubtful procedure, for we would then have to face the apparent contradiction that an autonomous increase in imports, clearly implying an increase of the particular country's total (and average) propensity to import, would not involve any change in its marginal propensity to import. The acceptance of the assumption may however be tolerated on these grounds: Firstly, an increase in total import demand may result in an increased, decreased or unchanged marginal propensity to import; and since there is no *a priori* reason for assuming a change upward or downward, the assumption of stability is not unreasonable. Secondly, if multiplier theory is to have any predictive value, propensities must either be stable or their rules of change must be known. With this vested interest in stable marginal propensities, one may leniently assume that slight changes in the total import propensity will not noticeably affect the "given" marginal propensity to import.

If a demand curve for imports as a function of income is drawn (as was suggested to me by Professor Arthur Smithies), an *autonomous* import increase implies a *shift* of this, upward sloping curve, while induced changes in imports are, of course, movements along the curve. If imports are plotted along the abscissa and income along the ordinate, an autonomous increase in the import demand—due to a change in tastes or a reduction in tariffs—would be shown by a shift of the curve to the right. (See Fig. 1.) If imports were plotted along the ordinate and income along the abscissa, the shift representing the autonomous increase in demand would be upward.

Fig. 1.—Demand for imports in terms of income.

The assumption of a stable marginal import propensity is expressed graphically by the facts (*a*) that the slope of the demand curve for imports (at

given incomes) is not changed when the curve shifts, i.e., the shifts must be parallel, and (*b*) that the curves are linear at least in the ranges within which income is induced to change in the course of the processes described by multiplier theory.

Such linear relationships between national income and imports were actually produced by Mr. Colin Clark (*The National Income of Australia*, p. 99). Of course, these linear functions could be said to isolate the income effects on the demand for imports only if all other influences upon imports were strictly ruled out, that is to say, if tastes, prices (domestic as well as foreign), tariffs and transport costs, etc., were unchanged throughout the period for which the function is calculated. Mr. Clark showed three such linear relationships between income and imports, one for the period 1922–27, another for 1928–30, a third for 1930–38; he attributed the "shifts" to autonomous import replacements due to tariff increases. While the shift from his second to his third function was parallel, thus leaving the marginal propensity to import unchanged, the shift from the first function was not a parallel one. A *reduction* of the total and average propensity to import was here associated with an *increase* in the marginal propensity to import— at least according to Mr. Clark's statistics and interpretation.

CONCERNING ASSUMPTION (2):

The assumption of stable prices, which we tried to justify above (p. 12), must of course be dropped before the theory is applied. We shall repeat this warning on several occasions and state it carefully in Chapter X. Commodity prices, foreign-exchange rates and interest rates are all likely to change to some extent in consequence of the change in the trade balance and, as a rule, are all pulling for its reversal. Nevertheless, the separation of income effects from price effects, by ruling out the latter provisionally, is methodologically sound.

CONCERNING ASSUMPTION (3):

The difficulty with the assumption that home investment is not affected is three-fold. There are, first, the changes in working capital employed in industries which produce exports or substitutes for imports; there is, second, the probable change in investment opportunities for fixed capital in these industries; and there is, third, the possible change in investment opportunities in domestic industries affected by the induced income changes. Nevertheless, clarity of exposition demands separation of the multiplier principle from all induced changes in home investment.

It is interesting to note in this connection that Professor P. T. Ellsworth in his *International Economics* (New York, 1937, p. 335) concludes that induced investment is the *only* lasting effective contribution to income that can result from an export increase. His reasoning, however, is defective and his conclusions are not borne out by the present study.

Mr. Metzler in his article on "The transfer problem reconsidered" chooses to include induced home investment in his model sequences by assuming a constant "marginal propensity to invest" (*loc. cit.* p. 401). Such an assumption would not change anything in our model sequences; the results would be analogous to those obtained if a higher marginal propensity to consume were assumed. The leakage through induced saving would be smaller if induced investment entered as a partly offsetting factor. I chose to rule out all induced home investment because I considered a *constant* marginal propensity to invest far less possible than constant marginal propensities to consume and to import. If investment behaves so "independently" as I believe it does, it is preferable to rule it out from a theory which works with constant propensities.

Many other simplifying assumptions are made for particular model sequences. Some of these assumptions are often rather fantastic but helpful in concentrating thoughts on specific points. For instance, in some cases (Models I, III and IX) induced saving is ruled out, in order to reveal the effects of induced imports (or exports, respectively) in splendid isolation. In two cases (Models I and II) foreign repercussions are ruled out, simply in order to contrast the effect of neglecting them with the results of including them in the analysis. In all models where two or more countries are looked at in order to study the cross-effects of the mutual repercussions, the assumption is made that monetary systems and habits of payments are so similar in the countries concerned that the marginal income propagation periods are everywhere of equal length. (The marginal income propagation period is the average interval of time which elapses from the receipt of an additional income by one set of persons until that portion of it which they spend on domestic goods and services is received as

income by another set of persons.)[1]   And all countries in the
multi-country models are assumed to be of equal relative
importance as to their shares in the exports and imports of all
other countries.

For fear of becoming tedious, this preview of the simplifying
assumptions is not made more detailed.   It is hoped, however,
that those which are not explicitly stated either here or at the
introduction of each model sequence will be clear and reason-
able enough to excuse their implicitness.

The standard problem treated in the model sequences in the
analysis of the autonomous-foreign-trade multiplier (Models
I–VII), is the following: Starting from a position of domestic
and international equilibrium (with unemployment, of course),
country A experiences an autonomous increase of exports to
country B.   Regarding this new export, of (let us say) green
cheese, two things are assumed: first, that its production does
not require any imported materials,[2] second, that it (i.e., the
new export) will be maintained henceforth, that is, it will
not be affected by any other changes in the foreign balance,
by income changes, or any other repercussions.[3]   The new
demand for country A's cheese in country B may have resulted
from a variety of causes: from a sudden change in the basic
diet of the B-people or from a reduction in the B-tariff on
cheese or from an export subsidy for cheese in country A.
The new import into country B is called an "autonomous"
change, since it was not induced by any change in B's income.
(The demand for foreign cheese in B is apparently of an
income-elasticity of zero, for from the assumption that the

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[1] See my article "Period analysis and multiplier theory," *Quarterly Journal
of Economics*, Vol. 54, 1939–40, p. 10.

[2] This assumption simplifies the exposition in that it avoids the necessity
of correcting the figures of autonomous exports.   Direct export-induced
imports would have to be deducted from autonomous "gross exports."

[3] This is a rather arbitrary assumption, to be sure, but sustained rates of
primary disbursement are essential for all multiplier models.

new export will be maintained it follows that the subsequent changes in B's imports, induced through changes in B's income, involve all kinds of articles except the extraordinary green cheese.)

The chief question to be answered will concern the changes in income per period which will be caused, in countries A and B (and perhaps in other countries), in consequence of the new cheese export. The ratios of the ultimate changes in income (in A and B, separately) to the value of the autonomous new export are the foreign-trade multipliers.

That the argument will run in terms of an export increase whereas arguments for protection would usually run in terms of import reductions should not suggest a difference which in fact does not exist. As long as all mentioned assumptions are maintained, the two cases of autonomous "improvement" of A's trade balance (i.e by reduction of imports and by increase in exports) will have the same results. If some autonomous shift in demand (for instance, one that is due to higher tariffs) turns expenditures of A-nationals from imported to domestically produced goods, the same processes will set in as are about to be described in the models which take an export increase as the autonomous change.[1]

MODEL I: NO INDUCED SAVING, NO FOREIGN REPERCUSSIONS

In order to isolate the leakage through imports, we are ruling out here the leakage through saving. This can be done simply by assuming that the marginal propensity to save is zero in country A. Thus, an increase in income in A will induce people to buy more imported as well as more domestic goods and services, but not to save any more than before.[2]

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[1] The problem "Import Restriction versus Export Increase" is discussed below on p. 58.

[2] There are other ways of ruling out the leakage through saving. For

We are ruling out, furthermore, all repercussions from induced changes in foreign countries. In previous discussions of the multiplier principle, foreign repercussions were usually not "ruled out" but simply overlooked.[1] To neglect foreign repercussions, instead of stating explicitly the assumptions under which there would not be any such repercussions, is hardly justifiable. If it is held that an autonomous increase of exports from A to B will increase incomes in A, one must not forget to think also about the decrease of incomes in B, and must not fail to study the effects which this income decline in B is apt to have upon A's export trade. But for the present model we assume that there will be no such foreign repercussions. Assumptions explaining the absence of repercussions (from B upon A) may be that the fall of incomes in B is prevented by a deliberate "offsetting policy" by B's fiscal and monetary authorities (with the offsetting measures becoming effective without any delay); or, that the marginal propensity to import is zero in country B, so that in spite of an income decline there would be no induced decrease in B's imports.

With no leakage through saving and no foreign repercussions, and with home investment absolutely unchanged, it is

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instance, we might admit that induced saving will arise with increased incomes, but we could assume that home investment rises immediately to the level of increased thrift, either because the marginal efficiency of investment increases (owing to the boom in the cheese industry or to the rise in consumption demand) just enough to take care of the higher level of intentional savings, or because interest rates fall (perhaps owing to the gold inflow which results from the cheese export) just enough to bring forth the needed investment activities. But we prefer here to hold investment incentives, interest rates and home investment absolutely constant, because this is what we shall want to assume in all other models of our analysis. Hence, for the sake of uniformity, we prefer to accomplish the elimination of the "leakage through saving" by assuming that the marginal propensity to save is zero.

[1] A notable exception is William A. Salant's excellent essay "Foreign trade policy in the business cycle," in *Public Policy*, Vol. II, published by the Harvard University Graduate School of Public Administration, Cambridge, 1941.

just the leakage through induced imports which will keep the income in country A (resulting from exports and re-spending) from growing to infinity.   Let us assume that the marginal propensity to import is equal to $\frac{3}{10}$, that is, that three tenths of an increment in income will be spent on increased purchases of imported goods and services.   If now an additional export of green cheese creates a primary income of $100.00 per period, the secondary income during the second period will be $70.00, the other $30.00 being spent for new imports.   The total income, primary plus secondary, resulting from the (continually repeated) green cheese export, will thus be $170.00 in the second period.   Each successive period will see a higher level of income, although the subsequent increments of income will soon become smaller and, after some time, negligible. In the seventh period, the income due to the steady green cheese export will have risen to $305.89, that is within 10 per cent of the ultimate equilibrium level of $333.33 per period.

What does the "seventh period" mean in terms of weeks, months or years? How should our operational time be translated into calendar time?   In a previous study I have estimated that an income-propagation period in the United States is an interval of about three months.[1]   If this estimate is correct, and I shall assume that it is, the "seventh period" means the time after some 18–21 months.   My estimate of the income-propagation period was based upon the theory that the interval between two successive income receipts is determined by the sum of the delay between income receipt and spending and the time needed for the spent money to pass through the hands or accounts of a number of firms (retailer—dealer—manufacturer of consumers' goods—various manufacturers of producers' goods and materials) and to reach eventually another income recipient.   For the actual quantitative estimate, data on the income velocity of circulation, corrected for the amount of entirely inactive money balances, were used.—Professor J. M. Clark, in an earlier study, estimated that the relevant period was only 2 months.[2]   However, he took account only of the delay between income

[1] "Period analysis and multiplier theory," *loc. cit.*
[2] J. M. Clark, *Economics of Planning Public Works*, Washington, D. C., 1935, p. 87.

receipt and spending, and not of the delay between spending and income receipt. Professor Alvin H. Hansen seems to accept Professor Clark's estimate.[1]—Professor Henry H. Villard, in an independent estimate, arrived at $3\frac{1}{2}$ months as the probable length of the income-propagation period.[2]—Professor A. C. Pigou, discussing a similar problem, estimated that the relevant period, which he called the "circulating period," was 6 months; he based his estimate also on data about the income velocity of circulation, but neglected to correct them for the portions of balances which are entirely inactive.[3]—Professor James W. Angell, reasoning in terms of an "average circulation period of active money alone" and using figures from 1899 to 1929, computed that the length of the period was between 3.15 and 3.33 months.[4]

The series of successive spendings can be followed up to the eleventh period (that is over a time interval of something like 30–33 months) in the adjoining Tables I-a and I-b. The symbols used in these tables and in those which will accompany the demonstration of other models have the following, fairly customary, meaning. Y stands for income, because I is reserved for investment. M stands for import (m-port), X for export (x-port),[5] S for saving and C for consumption. The sign Δ stands for the difference by which a particular magnitude exceeds, or falls short of, the *initial* level (not, as one might think, the level of the immediately preceding period). The symbol m stands for the marginal propensity to import, which in this example is $\frac{3}{10}$.

Tables I-a and I-b refer to one and the same sequence, only that the former is more elaborate. Table I-a gives a close-up

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[1] Alvin H. Hansen, *Fiscal Policy and Business Cycles*, New York, 1941, p. 240.

[2] Henry H. Villard, *Deficit Spending and the National Income*, New York, 1941, p. 256.

[3] A. C. Pigou, *Industrial Fluctuations*, London, 1927, pp. 136 ff.

[4] James W. Angell, *Investment and Business Cycles*, New York, 1941, p. 145.

[5] See D. H. Robertson's note, "Mr. Clark and the foreign trade multiplier," *Economic Journal*, Vol. 49, June 1939, p. 354, and Gottfried von Haberler, *Prosperity and Depression*, third revised edition, Geneva and New York 1941, p. 463.

view of the leakage, the gradual dwindling of the series of re-spendings starting from each dose of new income at its primary creation down to phases near the vanishing point.

TABLE I-a

AUTONOMOUS EXPORT OF COUNTRY A

Induced Imports. No Saving. No Foreign Repercussions

$m = 0.3$

Period		Income Recipients											Contributions to	
	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	r	M
1 Δr	100.00											100.00	
ΔM													
2 Δr	100.00	70.00										170.00	
ΔM		30.00											30.00
3 Δr	100.00	70.00	49.00									219.00	
ΔM		30.00	21.00										51.00
4 Δr	100.00	70.00	49.00	34.30								253.30	
ΔM		30.00	21.00	14.70									65.70
5 Δr	100.00	70.00	49.00	34.30	24.01							277.31	
ΔM		30.00	21.00	14.70	10.29								75.99
6 Δr	100.00	70.00	49.00	34.30	24.01	16.81						294.12	
ΔM		30.00	21.00	14.70	10.29	7.20							83.19
7 Δr	100.00	70.00	49.00	34.30	24.01	16.81	11.77					305.89	
ΔM		30.00	21.00	14.70	10.29	7.20	5.04						88.23
8 Δr	100.00	70.00	49.00	34.30	24.01	16.81	11.77	8.24				314.13	
ΔM		30.00	21.00	14.70	10.29	7.20	5.04	3.53					91.76
9 Δr	100.00	70.00	49.00	34.30	24.01	16.81	11.77	8.24	5.77			319.90	
ΔM		30.00	21.00	14.70	10.29	7.20	5.04	3.53	2.47				94.23
10 Δr	100.00	70.00	49.00	34.30	24.01	16.81	11.77	8.24	5.77	4.04		323.94	
ΔM		30.00	21.00	14.70	10.29	7.20	5.04	3.53	2.47	1.73			95.96
11 Δr	100.00	70.00	49.00	34.30	24.01	16.81	11.77	8.24	5.77	4.04	2.83	326.77	
ΔM		30.00	21.00	14.70	10.29	7.20	5.04	3.53	2.47	1.73	1.21		97.17

This elaborate table[1] has the pedagogic advantage that it shows, first, how an income flow increases as an aggregate while its parts are dwindling and, second, how the income flow begins to be stable only when the income particles created in the first period have all but vanished (because then periodic

COUNTRY A :						S = 0.0	M = 0.3
1	2	3	4	5	6	7	8
K = 3.33			Foreign Balance				Income from 2 + 6
Period	Spendable Income						
	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Auton. ΔX	Net Δ(X·M)	Current ΔY
1						100.00	100.00	100.00
2	100.00	70.00		30.00		100.00	70.00	170.00
3	170.00	119.00		51.00		100.00	49.00	219.00
4	219.00	153.30		65.70		100.00	34.30	253.30
5	253.30	177.31		75.99		100.00	24.01	277.31
6	277.31	194.12		83.19		100.00	16.81	294.12
7	294.12	205.89		88.23		100.00	11.77	305.89
8	305.89	214.13		91.76		100.00	8.24	314.13
9	314.13	219.90		94.23		100.00	5.77	319.90
10	319.90	223.94		95.96		100.00	4.04	323.94
11	323.94	226.77		97.17		100.00	2.83	326.77
12	326.77	228.75		98.02		100.00	1.98	328.75
13	328.75	230.14		98.61		100.00	1.39	330.14
14	330.14	231.11		99.03		100.00	.97	331.11
∞	333.33	233.33		100.00		100.00	0	333.33

TABLE I·b.—The same in changed presentation.

"creation" and "disappearance" of income will balance each other).

Table I-b has the advantage of taking less space. It will, therefore, be in this form that all other model sequences will be presented. Each period begins with the disposable income, received in the preceding period, and shows how this income is

[1] An analogous table, with the leakage through saving instead of the leakage through imports, was exhibited in my aforementioned article, *loc. cit.*, p. 18.

used.   The portion spent for imports is an item among those making up the foreign balance.   The current income of each period can be computed either by adding the portion of the disposable income which is spent on domestic consumers' goods, ΔC, (creating secondary income) to the autonomous export (creating primary income), or, alternatively, by adding the net-foreign balance of the period to the income of the previous period.

Both tables exhibit clearly the gradual increase of *induced imports*.   Already in the third period induced imports are $51.00, so that the primary contribution of $100.00 which the cheese export makes to the period's income is half offset; in this period only $49.00 of the export will be "expansionary" in character.   In the eighth period induced imports will stand at $91.76, that is within 10 per cent of the ultimate equilibrium level.   This ultimate equilibrium level, in the present case, will be $100.00, that is to say, equilibrium will be reached when induced imports become equal to autonomous exports, so that the expansionary effect of the exports vanishes and income stops rising.   As long as the exports exceed the imports, income continues to rise; and, in turn, as long as income rises, induced imports continue to rise; when induced imports reach the volume of the autonomous exports, income reaches its final level.   The ultimate income (per period) which is created by the autonomous, periodically repeated exports is a certain multiple of these exports, the multiplier in the present case being 3.33+.

This multiplier can be obtained by the formula $k = \dfrac{1}{s + m + f}$, where k denotes the multiplier, s the marginal propensity to save $\left(\dfrac{\Delta S_t}{\Delta Y_{t-1}}\right)$, m the marginal propensity to import $\left(\dfrac{\Delta M_t}{\Delta Y_{t-1}}\right)$, and f the foreign repercussion factor.   Since

s and f are zero in the present case, and m is .3, $k = \dfrac{1}{.3}$ or 3.33+.

THE LAG OF IMPORTS BEHIND EXPORTS

The fact should be emphasized that the *gradual* rise of induced imports has an important function in this whole multiplier business. Money income rises only because, and as long as, induced imports fall short of autonomous exports. Only the lag of imports behind exports makes it possible that money income rises as a consequence of the exports.[1] If,

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[1] In order to safeguard this statement against possible criticism, it seems necessary to consider various possibilities in the "timing" of exports, incomes and imports, not all of which would conform to the model just described. (1) If exports start unexpectedly so that the first deliveries are made from inventories (thus, resulting in net disinvestments) incomes do not increase with the exports in the first period. (We must bear in mind that sales proceeds of firms are disinvestors' receipts rather than income; see above p. 2.) Only increased production (i.e. re-investment to replenish the reduced inventories) will raise incomes. Exports out of commodity stocks represent domestic disinvestment; the exporters will hold more cash in the place of inventory, or they will repay bank loans. Only when the production of the export article is adapted to the export demand will exports create income. If the reduced commodity stocks are replenished through more production, the failure of the exports of the first period to create income will be made up later.

(2) If export orders are received sufficiently ahead of delivery, the increase in production may precede the export. To finance the increase in production producers will either borrow from the banks or use idle cash reserves; this constitutes increased domestic investment. Incomes will thus be increased before the exports take place. When deliveries are made and payments for the exports are received, the financing of the stepped-up production will be taken over by the proceeds from abroad, with the bank credits initially incurred or the cash balances initially dishoarded becoming a revolving fund. That the income increase preceded the export was clearly the result of home investment. And, of course, in this case imports need not lag behind exports and yet the higher level of income may be sustained.

(3) Both previous cases were contrary to our assumption that home investment would not change. We may prevent the disinvestment seen in (1) and the investment seen in (2) by assuming that the exporters receive the foreign funds when they are producing the export articles and paying domestic funds to the employed factors of production. In this case the export, the receipt of the foreign funds and the disbursement of the domestic

through some mysterious spell, imports (paid out of received incomes) jumped to the volume of exports without any lag at all, there would be no expansionary force in our cheese exports. But this is not what takes place in our model; in our model, M will be equal to X in the eventual equilibrium; this equilibrium comes about only after a period of adjustment of income, that is, it is bound up with an increase of Y.

One cannot help being reminded of the analogous phenomena of autonomous investment, I, and induced savings, S, the latter being functionally related to a rise in income, Y. Unfortunately, the now fashionable definition of saving, which makes it identical with investment, prevents many students from understanding the function of a lag of S behind I. Using (more easily comprehended) definitions where saving refers to previously (rather than simultaneously) received income, one can conveniently say that the excess of investment

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funds are synchronized, whereas the re-spending of the domestic funds comes afterwards. This corresponds to our model.

(4) One may imagine that export orders are received ahead of time and, in anticipation of increased incomes, foreign goods are ordered in the full value of the exports on order. If we want to avoid the assumption of increased home investment we may either stipulate that the factors of production accept the increased incomes in kind, that is in imported goods, or that consumers, anticipating higher incomes, use idle cash reserves or consumers' credit to pay for imported goods before they receive pay for their participation in the production of export articles. In these cases imports need not lag behind exports for incomes to be increased by the full amount of the exports. But the assumptions are rather peculiar.

(5) By making still more fantastic assumptions one may even get a multiplier effect (in excess of $k = 1$) without the import lag. If everybody who is going to get an income knows it well in advance and, in anticipation of it, buys domestic and imported products, paying for them either by dishoarding idle cash or procuring consumers' credit, then all increased production for export as well as for domestic consumption may be financed from these sources. In this case income might exceed the exports without imports lagging, and what otherwise would take almost two years might be accomplished "instantaneously."

If the assumptions employed in (4) and (5) are rejected and if it is assumed instead that people generally buy out of received income, the statement in the text will hold true: the lag of imports behind exports makes it possible that money income rises in consequence of the exports.

over saving creates higher income levels, and that higher income levels induce more saving, until (induced) savings, after some time, become equal to (autonomous) investment, and equilibrium is established at a higher level of income.

Fortunately, nobody has as yet tried to make imports by definition equal to exports. The equality of (induced) imports to (autonomous) exports is reached only after an adjustment period, and only then is equilibrium established, at a higher level of income. Y rises because, and as long as, $\Delta X > \Delta M$; in equilibrium $\Delta X = \Delta M$, and Y is increased as against its initial level by an amount which is related to ΔX by the multiplier; this multiplier is determined by m, or the force which, with increasing Y, induces ΔM to approach ΔX.

If appropriate definitions permit us to treat induced saving analogously to the fashion in which we treat induced imports, the understanding of other models in which induced saving is no longer ruled out by assumption, will be much facilitated. For this reason we shall find it more convenient (though of course, not indispensable) to work with the good old concept of saving as the nonconsumed portion of the just previously received income.[1] This conceptual scheme will enable us to think of income as increasing because $I + X > S + M$, and of equilibrium as being established when $S + M$ becomes equal to $I + X$.

CLASSICAL FOREIGN TRADE THEORY

Before we proceed to models which include induced saving, we may take time out for a brief digression on the role of induced imports in classical foreign-trade theory. The theory that increased exports would expand the money flow, and that

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[1] "Just previously" refers to the "preceding income period" of the saver; in our analysis of aggregates, we take the national income propagation period as the convenient time interval to which the concept of "saving out of received income" is to refer.

this expanded money flow would induce imports, was, of course, classical theory *par excellence;* only that classical writers were more interested in the long-run equilibrium itself than in the increased money flow through which the equilibrium came about.   To classical writers the increased money flow was merely a medium of adjustment, not an end to be desired, particularly because the temporary expansionary effects could not raise (least of all, permanently raise) the real national income in an economy in which resources were assumed to be fully employed.

One may recall the famous controversy among various classical (and also modern) writers on the question whether the rectification of the trade balance would come about merely through relative shifts in the amounts of money in the exporting and importing countries, that is, through changes in money income, or whether relative price changes would also be necessary.   Some of the writers held that income changes were sufficient, others that price changes were necessary, to rectify the trade balance.[1]   Modern multiplier theory concentrates on income changes; in Model I, the income effects were sufficient for an equilibration of the trade balance.   But this will not be so in other models.

It is interesting to read the exposition which the theory of income-induced imports received at the hand of one classical writer, Mountiford Longfield.   In an article on" Banking and Currency," published in 1840,[2] Longfield discussed the case of a suddenly increased grain export from "Prussia or Russia" to "England."   (Prussia or Russia are, thus, our country *A*, England is country *B*.)   Will, Longfield asks, the imports of

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[1] For references concerning this controversy see Gottfried von Haberler, *The Theory of International Trade*, London, 1936, pp. 66–76, and Jacob Viner, *Studies in the Theory of International Trade*, New York, 1937, Ch. VI, pp. 290–387.

[2] *Dublin University Magazine*, Vol. XV and XVI, quoted by Jacob Viner, *op. cit.*, pp. 297 and 620.

the grain-exporting countries immediately rise to the level of
the increased exports?

"Will the Prussian or Russian landowner, whose wealth has been sud-
denly increased, be content to expend his increased wealth in the purchase
of an increased amount of English manufactures? We say that the con-
trary will take place, and that his habits will remain unchanged, and *his
increase of wealth will be spent in nearly the same manner as his former income*, that
is to say, not one fiftieth part in the purchase of English goods. *His country-
men will, in the first instance, have the advantage of his increased expenditure.* It will
not be felt in England until after a long time, and passing through many
channels. . . . "[1]

Thus, Longfield assumed that the propensity to import was
rather low and that it was stable, so stable indeed that the
marginal propensity to import was equal to the *average* propen-
sity to import. And, furthermore, Longfield expected that,
owing to the high propensity to consume home products, the
income of the exporting country would be favorably affected.
The similarity between Longfield's exposition and more recent
discussions, rather striking up to this point, ends here because
Longfield proceeded to introduce price increases in the export-
ing country and price declines in the importing country.

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[1] *Loc. cit.*, Vol. XV, p. 10, quoted from Viner, *op. cit.*, p. 297. (Italics are
mine.)

Chapter III

THE SECOND MODEL: INDUCED IMPORTS AND SAVING. STATISTICAL MULTIPLIERS. TIME AS A VARIABLE

MODEL II: INDUCED SAVING, NO FOREIGN REPERCUSSIONS

Let us proceed to other model sequences. The leakage through saving, which we ruled out of the first model, will be re-admitted in the second model; here it will be demonstrated how this leakage works together with the leakage through imports. Foreign repercussions, however, continue to be excluded in Model II, that is to say, the cheese import into country B still will not cause any repercussions which could affect the export trade of A.

Let us assume that, in country A, the marginal propensity to save is $\frac{2}{10}$ and the marginal propensity to import is again (as in Model I) $\frac{3}{10}$. With these leakages totaling $\frac{5}{10}$, or $\frac{1}{2}$, of any income increments, the full multiplier will be found to be 2. Hence, a steady export of $100.00 worth of goods or services per period will ultimately result in an income level which will exceed the initial one by $200.00 per period.

Table II follows the series of successive spendings up to the ninth period, that is, through a time interval of something like 24–27 months assuming an income propagation period of about three months. We see that the $100.00 which the first income recipients (probably people who have something to do with the production of green cheese) take in, are used in the

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following fashion: $20.00 are saved, $30.00 are spent on increased purchases of imported goods or services, and $50.00 are spent on increased purchases of domestic goods and services. These $50.00 are secondary incomes and are used by their recipients in the same way, that is, the same proportions being saved and spent for foreign and domestic purchases, respectively. Already in the fourth period—hence within 9 or 12 months—income will have increased by $187.50, i.e., within 10 per cent of the ultimate $200.00.

COUNTRY A :					S=0.2	M=0.3	
1	2	3	4	5	6	7	8
K=2			Foreign Balance				income from 2+6
Spendable Income							
Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Auton. ΔX	Net Δ(X-M)	Current ΔY
1					100.00	100.00	100.00
2	100.00	50.00	20.00	30.00	100.00	70.00	150.00
3	150.00	75.00	30.00	45.00	100.00	55.00	175.00
4	175.00	87.50	35.00	52.50	100.00	47.50	187.50
5	187.50	93.75	37.50	56.25	100.00	43.75	193.75
6	193.75	96.87	38.75	58.12	100.00	41.88	196.88
7	196.88	98.44	39.38	59.06	100.00	40.94	198.44
8	198.44	99.22	39.69	59.53	100.00	40.47	199.24
9	199.24	99.62	39.85	59.77	100.00	40.23	199.62
∞	200.00	100.00	40.00	60.00	100.00	40.00	200.00

TABLE II.—Autonomous export of country A. Induced saving and imports. No foreign repercussions.

Induced savings rise from $20.00 in the second period to $37.50 in the fifth period, and then slowly further to an ultimate $40.00 per period. Induced imports rise from $30.00 in the second period to $56.25 in the fifth period, and then slowly on to an ultimate $60.00. (It will be apparent that the final level of induced saving must be two tenths, and the final level of induced imports three tenths, of the final income increase. On the other hand, it should be obvious that the words "final" and "ultimate" are used somewhat loosely in connection with an endless progression. But for

practical purposes we may forget that the last penny will never be quite "full.")

The autonomous export creates income, and this new income induces saving as well as imports. As long as saving and imports together lag behind the autonomous exports, income continues to rise, causing a continued growth of induced saving and induced imports. When saving and imports catch up with the exports, income will have reached its new equilibrium level. Substituting arithmetic values in the equation describing the equilibrium conditions, $\Delta I + \Delta X = \Delta S + \Delta M$, we write $0 + 100 = 40 + 60$, with the understanding that the left side of this equation contains the autonomous items, the right side the induced ones. The income growth comes to its end when the current contributions to income (made, in this case, by the cheese export) are fully offset by induced saving and induced import.

INDUCED FOREIGN LENDING

Looking at the net change in the trade balance in the new equilibrium position, we find that against the new exports, amounting to $100.00 per period, there are induced imports in an amount of $60.00, making for an active foreign balance of $40.00. In contradistinction to Model I, equilibrium involves here not an equality of exports and imports, but a regularly recurring export surplus. This export surplus, however, is not of an expansionary (i.e., net income creating) character, because it is offset by saving. With home investment assumed to be unchanged, the new savings, in fact, are financing the export surplus. The new savings find their realization in foreign lending[1] or, otherwise, in gold imports.

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[1] In order to avoid misunderstanding it should be noted that foreign lending, in accepted terminology, does not mean lending by foreigners but means lending *to* foreigners.

This result may seem bewildering at first sight. Why should people feel a sudden urge for making foreign loans? It was easy to comprehend how the cheese export led to additional saving; but why should the cheese export induce these savings to be invested in "foreign lending"?

Of course, one must not think that the savers themselves are making foreign loans or buying foreign securities. To be sure, this would not be impossible. The increased supply of investible funds, with an unchanged home demand for them, may well seek placement in assets held by foreigners—especially if (as will be the case in Model IV and later ones) induced dissavings abroad may result in a supply of foreign-held securities which just "meets" the new supply of investible funds in the country favored by the export surplus. To the extent that this purchase of hitherto foreign-held securities takes place, one may perhaps speak of an induced international movement of "long-term capital" (for, by a widely accepted convention, international shifts in securities holdings are called long-term capital movements, even if the shift is only temporary).

However, one may well consider it too strange a coincidence if the induced savers were also induced to place their funds abroad or in hitherto foreign-held securities. Other developments would be more likely. The savers may accumulate idle balances; or they may make domestic loans or acquire domestic securities. But, since the domestic demand for loanable funds, or domestic supply of securities, has not changed, the savers in their capacity as lenders or securities purchasers would "steal" earning assets from the banking system. But who makes the foreign loans? They are made, in all likelihood, by that same banking system, especially if the "banking system" is defined broadly enough to include the "monetary authority." For, if foreign-exchange rates are kept stable (as we assumed) in spite of the export surplus and in

absence of private loans to or investments in foreign countries, one of the following three things must happen: (1) Banks or the monetary authority must acquire foreign balances; (2) foreign deposits in our banks must go down; or (3) banks or the monetary authority must acquire foreign gold. The first two of these possibilities constitute a change in the amount of foreign indebtedness, a change which may be briefly called "net foreign lending." Hence, the new savings are invested in (or counterbalanced by) either foreign lending or gold imports.

We can now see what may have been meant by certain modern writers when they claimed that neo-classical economists were mistaken in their belief in a "one-way-causality" from foreign lending to export surplus, and never the other way around, from export surplus to foreign lending. According to neo-classical theory, the balance of trade was the "induced" phenomenon, the capital movement the autonomous one.[1] Modern dissenters, often referred to as Neo-Mercantilists, claimed that the opposite relationship prevailed, with a change in the trade balance as "cause," or autonomous phenomenon, and a capital movement as "effect," or induced phenomenon.[2]

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[1] When we speak now of the "induced" phenomenon—and of "induced" foreign lending—we must note that "induced" is not used here in the same sense in which it was used in connection with saving or imports (namely in the sense of "induced by an income change") but instead in a looser sense of assigning the place in a general cause-and-effect relationship.

[2] Thus, Mr. J. M. Keynes maintained that even long-term capital movements adjusted themselves to the trade balances rather than the other way around. See his article on "The German transfer problem," *Economic Journal*, Vol. 39, 1929. For a pointed discussion of this issue see Jacob Viner, *Studies in the Theory of International Trade*, p. 364. See also Gottfried von Haberler, *Prosperity and Depression*, Third Edition, p. 472. P. T. Ellsworth, in his *International Economics*, p. 333, makes the following, fairly typical statement: "Unless *some outside force* such as an increase of foreign lending permits their continuance, exports must also be expected to shrink by a like amount." (Italics are mine.) I interpret an "outside force" to be an "autonomous" one. A detailed discussion of these problems will be found in Chapter VIII below.

The unmitigated classicist objects to the neo-mercantilistic theory, on the ground that the well-known monetary mechanisms—the specie-flow mechanism, the self-adjustment of the balance of payments—would force a quick reversal of the movements. Foreign lending, according to this view, can not go on for a long time unless it is autonomous, that is, unless it is a flow of long-term investment into foreign countries stimulated by factors independent of the current state of the balance of payments. Induced foreign lending and induced gold imports, on the other hand, would expand domestic monetary circulation and, in due time, result in increased imports and decreased exports—until the favorable trade balance would have disappeared. The (modern) answer to this (classical) argument is now that neither gold inflow nor induced foreign lending by the banking system is net-expansionary if offset by increased thrift. The apparent expansion through the continuous export surplus is not a net expansion if it is absorbed by induced saving. Hence, gold inflow and induced foreign lending are not self-reversing but can go on forever, or, that is to say, can go on as long as the monetary authorities in the countries concerned stand for these consequences of their policy of stable exchange rates and stable interest rates.[1]

Granting the correctness of these conclusions, one may do well to remember the assumptions from which they were derived. Absence of foreign repercussions was one of the assumptions; but proper consideration of foreign repercussions will later be seen merely to qualify, not to invalidate, the neo-

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[1] "Orthodox" gold standard rules, calling for a central bank policy which regulated interest rates according to international gold movements, anticipated the theory of the non-automatic reversal of the balance of payment in that they super-imposed upon the automatic adjustment of income some degree of management through discount policy. The primary income changes, even multiplied by the foreign-trade multiplier, are possibly insufficient to force the reversal of the balance of payments and must be reinforced through secondary changes effected by "orthodox" central bank policy.

mercantilistic reasoning.   However, among the assumptions were also perfectly elastic supplies of products and of unemployed resources at given prices (and also a perfectly elastic supply of bank funds at given interest rates); proper consideration of inelasticities of supply may do serious damage to the theory in question.   In this analysis, however, we shall stick to our chosen assumptions and "shake out" all their hidden implications, no matter whether the outcome seems to give support or discomfort to either classical or mercantilistic ideas.

STATISTICAL MULTIPLIERS

In Model II an autonomous increase in exports in an amount of $100.00 is seen to produce a final increase in income in an amount of $200.00.   The multiplier, describing the effect which the autonomous change in trade has upon income, is 2.   If a statistician were to verify this theory by checking empirical data obtained from foreign trade statistics and income statistics, what would he find?

First of all, he would be faced with the problem of choosing a proper time lag.   The adjustment of income to an autonomous change in foreign trade is not immediate; it takes several income propagation periods (of perhaps 2, 3, or $3\frac{1}{2}$ months each) until income comes near enough to the level which corresponds to its "determinants."[1]   If the statistician tried a

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[1] The opposite view is held by Mr. Colin Clark in his article on the "Determination of the multiplier from national income statistics," *Economic Journal*, Vol. 48, 1938, p. 439: "The various stages of the readjustment take place in very quick succession (a very large part of the whole national income is always spent within a week of its being earned) and, thinking in terms of the three-months time unit which we are using, can be regarded as immediate."   Mr. Clark's "three-months time unit" refers to his use of quarterly figures; hence, he believes that quarterly statistics would show all magnitudes "readjusted" to one another; this obviously presupposes income propagation periods not much longer than the week which he mentions.   In his previous work, *The National Income of Australia*, p. 96, Mr. Clark had said literally: "*Several* rounds of expenditure will take place in a *few* weeks, and adjustment is, from an economic point of view, virtually *instantaneous*." (Italics are mine.)   Mr. Clark's error lies in his confusion between spending

time lag of five or six months and, thus, took the rate of income which was received before the third round of spending had started, he would find an income increase of $150.00— hence, a multiplier of 1½ instead of 2.   And he would proba- bly draw wrong conclusions concerning the relevant propen- sity to consume home-produced goods and services—if he were so ill-advised as to accept the methodological trick of deducing and measuring an alleged constant by means of the same data which it is supposed to explain.[1]

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out of income and receiving of income.   Even if large sums of received income are re-spent within a week, this does not imply that they arrive immediately at the next income recipient and are ready for another round of spending.   The flow of funds through several intermediate stopping stations in trade and industry takes time.

Mr. Clark neglected the readjustment period only for a "very large part of the whole national income," but was willing to assume a "delay" through the slow spending on the part of the rich.   For these "delayed effects of income change in altering the consumption-expenditures of the richer portion of the community" Mr. Clark has suggested (*Economic Journal*, pp. 439–40) a time lag of a year and a quarter.   Since the bulk of all saving and (according to Mr. Clark, though not in fact) the whole of marginal saving is done by these slow spenders, the *lag through slow spending* makes the ultimate *leakage through saving* smaller than the instantaneous leakage.

The assumption of this lag through delayed spending of course does not repair the damage from the neglect of the lag through delayed arrival of spent moneys at the next recipients of income.

[1] This is one of the most popular games in economics: first you name the force which you think can bring about a certain result; then you measure the strength of that alleged "force" by the magnitude of the observed "result"; and you wind up "testing" your theory by showing that the "expected" results and the actual results are the same.   In the particular case of the marginal propensity to consume as the constant which is sup- posed to explain the relationship between autonomous disbursements and national income, Professor Haberler has warned against the fundamental errors in reasoning.   (See Gottfried von Haberler, "Mr. Keynes' 'theory of the multiplier': A methodological criticism," *Zeitschrift für Nationalökonomie*, Vol. 7, 1936, pp. 299–305.)   Nevertheless there seems to be no end to the stream of victims of this snare.   A recent accident is Mr. Mordecai Ezekiel who, in his articles on "Saving, consumption, and investment," *American Economic Review*, Vol. 32, 1942, tries to "compute" from his data on invest- ment and national income not only the marginal propensity to save but also a marginal propensity to invest.

But, we must ask secondly, would the statistician be able to compare the observed increase in income with the autonomous increase in exports? How should he know which change in foreign trade was autonomous and which was induced? After all, we cannot expect reality to do us the favor of holding back all further autonomous changes until the effects of the last change are all complete. There is probably not one period, however short, in which there would not occur both induced changes and new autonomous changes in foreign trade. How should the two types of change be disentangled and identified?[1] In our example, in Table II, we knew which changes were autonomous and which induced—because we had made up the figures ourselves. When we saw that in the third period exports were up by $100 and imports were up by $45, we "knew" that the ones were autonomous and the others induced. But actual trade statistics do not tell the

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[1] Mr. Colin Clark (*The National Income of Australia*, p. 99) asserts that from 1922 to 1938 all changes in Australian exports were autonomous, and all changes in imports were income-induced, with the exception of two autonomous changes of imports in 1927 and 1930, respectively. In his "Comment," *Economic Journal*, Vol. 49, 1939, p. 356, he assures us: "It was very fortunate that the situation in Australia over the last ten years has been simple enough" to make the separation possible, "independent information being available for the tricky task of estimating the extent of the 'autonomous' movements of imports." How accommodating a nation are these Australians, how considerate of the ambitions of statisticians, that they hold their tastes, techniques, prices, etc., constant, or keep them from influencing the level of imports, over periods as long as eight years. Mr. Clark might at least have tried to justify his procedure by contending that the only *conspicuous* and *large* autonomous import changes were those two in 1927 and 1930 and that all other autonomous import changes were probably negligible.

Professor Viner, on practically the same issue, observed that "except when there are drastic disturbances whose origin is fairly obviously to be associated with contemporary events external to the mechanism of international trade itself, it will ordinarily be fruitless to try to distinguish disequilibrating from adjusting factors." (Jacob Viner, *Studies in the Theory of International Trade*, p. 364.) Professor Viner's "disequilibrating" corresponds broadly to our autonomous, "adjusting" to our induced changes. (Incidentally, in the original text of the above quotation it reads "equilibrating" instead of disequilibrating; this is obviously a misprint.)

"causes" of trade. Would the statistician not be forced to give up any attempt to distinguish between causes and effects? Would he not simply have to take indiscriminately the net foreign balance as his point of reference? Indeed, this is what actually was done in at least one published computation of the "multiplier."[1] If then the ultimate increase in income were compared with the ultimate foreign balance, the multiplier would not be 2. The improved trade balance, the result of autonomous exports and induced imports, would be $40, and this, in relation to an income growth of $200, would mean a multiplier of 5. Induced importing, a factor rightly belonging into the multiplier, would have been shoved back into the multiplicand. The multiplier would express merely induced saving. And, since the propensity to save was assumed to be .2 or ⅕, the corresponding multiplier would be 5.[2]

Yet even this result could be obtained only if the statistician were able to leave out all adjustment periods and to select nothing but times of full equilibrium. But, of course, an

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[1] Colin Clark, "The determination of the multiplier from national income statistics," *Economic Journal*, Vol. 48, 1938. Mr. Clark tried to correct his computed multiplier afterwards for the inclusion of induced ("consequential") imports in the multiplicand; but, apart from all arbitrariness in his assumptions, he fell into grave errors of reasoning. See the next footnote.

[2] Compare the remark by Mr. Colin Clark (*The National Income of Australia*, p. 92) "Some of the factors involved here would be regarded . . . as affecting a change in the multiplier itself. According to the present method, the multiplier is considered as constant in those circumstances, and the changes subsumed in the magnitude of the multiplicand."
It should be clear that a *smaller multiplicand* (autonomous exports *minus* induced imports) and a *given product* (the income increase) must mean a *larger multiplier*. Thus, somebody starting with the small multiplicand (small because the foreign balance is of course less than the autonomous exports) and intending to correct the multiplier for his failure to obtain the correct multiplicand, must *reduce* the multiplier which he computes from his data. Mr. Clark *raised* it instead (see "The determination of the multiplier from national income statistics." *Economic Journal*, Vol. 48, 1938, p. 442). Mr. Clark's error was discussed also by Professor Henry H. Villard, *Deficit Spending and the National Income*, pp. 173–75.

elimination of transition periods is practically impossible, for, in our world of change, we always live in transition periods. Imagine, then, that the statistician gets his data for a full year, that is, perhaps, for about the first four periods in Model II. The cumulative export surplus for that time is (100.00 + 70.00 + 55.00 + 47.50 =) $272.50, the cumulative income increase is (100.00 + 150.00 + 175.00 + 187.50 =) $612.50. Based on these figures, the multiplier would be 2.24. Taking, instead, the cumulative export surplus and income increase of two years (eight periods), the respective figures would be $360.46 and $1400.82, and the multiplier would be 3.91. None of these figures has any significant relationship to what one would expect if he applied the theory as it is usually stated, namely, with the "full" (ultimate) foreign-trade multiplier.

THE SEPARATION OF INDUCED FROM AUTONOMOUS CHANGES

Returning once more to the problem of the statistical separation of induced from autonomous changes in foreign trade, we must now go beyond the relatively simple task offered by our Model II, where all changes in exports were autonomous and all changes in imports were induced. We might just the same have started from an autonomous import reduction (rather than from the export increase) so that all subsequent changes in imports would have been partly autonomous and partly induced. The autonomous part of these changes in imports could be the result of duly recorded changes in tariffs or of less conspicuous changes in foreign or domestic prices or of entirely invisible changes in tastes. And these changes may be gradual rather than sudden, and, hence, still less identifiable. Or let us think for a moment of the scheme put forth by Mr. Harrod[1] in his exposition of the multiplier of home investment and export increases. There

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[1] R. F. Harrod, *The Trade Cycle*, Oxford, 1936, pp. 145–58.

the changes in imports are theoretically separated into (1) those directly resulting from the increased home investment (for example, foreign materials needed for the construction of the investment projects), (2) those directly resulting from the increased exports (for example, foreign materials needed for the production of the export articles), and (3) those resulting from the increased consumption demands due to the increments in incomes which were generated by the investments and autonomous exports (for example, foreign materials needed for the production of consumption goods). Now, the first two of these imports should be subtracted from the multiplicand, the third, the income-induced imports, belong into the multiplier. Mr. Harrod's multiplier formula takes appropriate account of this. But the task for the ambitious statistician becomes rather hopeless.

This is not all. It is absolutely untenable to regard all changes in exports as autonomous. One cannot possibly assume that most import changes are induced and all export changes are autonomous, for, after all, the induced import of one country must be some country's export and, thus, cannot be an autonomous export there. All of our later models will show exports affected by income changes abroad, changes which are consequential upon previous autonomous changes in foreign trade. Thus we shall become acquainted with induced increases and decreases of exports. And in some of our models (e.g. VI and VII) we shall even see that a country may experience changes of exports which are not consequential upon any autonomous changes in the foreign trade of that same country but are repercussions of changes in trade between two other countries. These exports will not qualify as autonomous changes.[1] Some of these foreign-income-induced exports have a multiplying effect far in excess of that

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[1] But Mr. Colin Clark is firm about it. "I still maintain that the most useful approach is that which treats investment, exports, autonomous imports and 'excessive spending' as determinants [multiplicands]" and the

of autonomous exports, but on the other hand, they may be less continuous and, in some cases, indeed ephemeral.[1]

Under these circumstances one might well be inclined to consider the cause of a valid statistical determination of the multiplier as lost because of technical difficulties—if it is not regarded as lost anyway because of probable instabilities of the psychological propensities. Propensities must be stable if the multiplier computed for one time is to have any use for application to situations at another time. But statisticians are not easily discouraged. They hope that the variations in propensities through time are not serious. They assume, for certain countries, that autonomous changes in imports cannot be important.[2] They hope that inconspicuous autonomous changes in imports can be neglected without seriously affecting the results obtained from following up the developments after the really big, easily separable changes, or, better still, that the smaller autonomous ups and downs in imports may be assumed to cancel out.[3] Or they expect to tackle all technical difficulties with the modern weapons of partial correlation analysis.[4]

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marginal propensities to consume and to import "as forming a multiplier." ("Comments," *loc. cit.*, p. 357.) This was in rejoinder to Professor D. H. Robertson's remark: "Mr. Clark has become uneasy, as well he may be, about the danger of mixing up cause and effect when dealing with the multiplier." ("Mr. Clark and the foreign trade multiplier," *Economic Journal*, Vol. 49, 1939, p. 355.)

[1] Like the exports of country C in Model VII.

[2] In the United States "imports are small in any case, and no autonomous changes in the level of imports can be of much importance," says Mr. Colin Clark, "Comment," p. 356.

[3] For this suggestion I am indebted to Mr. Milton Friedman.

[4] It is again the imperturbable Mr. Colin Clark who expressed the hope that "analysis by partial correlation" will "make possible the analysis of the infinitely complex situation of the actual world." ("Comment," p. 358.) Mr. Clark does not indicate how he will use partial correlation analysis for the solution of our particular difficulty. It should be noted that we do not suffer here from the large number of variables but from our inability to decompose one variable, the foreign balance, into components to which we ascribe different causal significance.

However optimistic or sceptical we are concerning the chances of obtaining satisfactory solutions of these statistical problems and acceptable computations of the "constants" involved in our theory, its explanatory and pedagogic value need not be affected either way.

THE MULTIPLIER AS A FUNCTION OF TIME

In recognition of the fact that the "full" multiplier can be attained only after an infinite length of time, one may find it desirable to arrive at formulas which give us (*a*) the income level attained after a time interval, *t*, has elapsed, and (*b*) the time interval which it takes for income to increase to a level within 10 per cent of the "full multiple."

Time may be measured here by the number of income propagation periods. The income increase of the first period must *ex hypothesi* be equal to the "autonomous" increase in home investment and export, since induced saving and induced imports are assumed to occur only *after* income has been received. Thus, using the same notations as before (but omitting, for the sake of brevity, the Δ signs),[1] $Y_{t=0} = 0$, and $Y_{t=1} = I + X$. The income increase according to the "full multiplier" will be denoted as $Y_{t=\infty}$ or simply Y_∞.

Our present tasks are then to find

(*a*) an expression for Y_t, and

(*b*) the value of *t* for which $Y_t = .9Y_\infty$.

Since we are concerned here with the foreign-trade multiplier we must rule out any home investment and confine the analysis to autonomous exports, *X*, as the only income-increasing expenditure. If only one country is considered and foreign repercussions upon that country's exports are ruled

[1] The income increase (over and above the level that prevails at time 0) is denoted as Y, additional investment as I, and additional exports as X. The marginal propensity to save is called s, the marginal propensity to import is called m, the marginal propensity to consume home-made goods is called c. Of course, $s + m + c = 1$, or $c = 1 - s - m$.

out, the solutions of our problems are simple. (If foreign repercussions are included, the equations will be complicated, as will be seen later.)

According to the customary multiplier formula,

$$(1) \qquad Y_\infty = X\frac{1}{s+m} = X\frac{1}{1-c}.$$

In order to determine the income increase for any period, we have to realize that this magnitude rises as the sum of a simple geometric progression:

$$(2) \qquad Y_t = X + cY_{t-1}.$$

Using the well-known summation formula for geometric progressions, we obtain

$$(3) \qquad Y_t = X\frac{1-c^t}{1-c}.$$

Searching for the value of t, i.e., the number of periods in which Y_t reaches 90 per cent of Y_∞, we write

$$(4) \qquad Y_t = .9Y_\infty;$$

after a few simple steps[1] we obtain

$$(5) \qquad t = \frac{\log .1}{\log c} = \frac{-1}{\log c}.$$

(Log c is negative because $c < 1$; hence t is always positive.)

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[1] Substituting (1) and (3) in (4), we get

$$(4a) \qquad X\frac{1-c^t}{1-c} = .9X\frac{1}{1-c}.$$

This reduces through cancellation to

$$(4b) \qquad 1 - c^t = .9$$

or

$$(4c) \qquad c^t = .1.$$

Solving for t, we write

$$(4d) \qquad t \log c = \log .1,$$

and, finally

$$(5) \qquad t = \frac{\log .1}{\log c}.$$

For the convenience of the reader the t's of the ninety-percent-full-multiples are given here for several values of c.

TABULATION GIVING THE TIME NECESSARY FOR THE INCOME INCREASE TO REACH
90% OF THE FULL MULTIPLE

Marginal Propensity to Consume Home-products	Logarithm of the Marginal Propensity to Consume Home-products	Number of Income Propagation Periods after which the Income Increase Reaches 90% of the "Full Multiple"	Approximate Number of Months
c	Log c	$t = \dfrac{-1}{\log c}$	1 Period = 3 Months
1.	0.	∞	∞
.9	− .0457575	21.85	65.6
.8	− .0969100	10.32	31.
.7	− .1549020	6.46	19.4
.6	− .2218487	4.51	13.5
.5	− .3010300	3.32	10.
.4	− .3979400	2.51	7.5
.3	− .5228787	1.91	5.7
.2	− .6989700	1.43	4.3
.1	−1.0000000	1.00	3.
.05	−1.3010300	.77	2.3

Chapter IV

THE THIRD MODEL: INDUCED FALL OF EXPORTS. THE SELF-ADJUSTMENT OF THE FOREIGN BALANCE. INVISIBLE ITEMS

MODEL III: FOREIGN REPERCUSSIONS, NO INDUCED SAVING

We are introducing now the repercussions to be expected from foreign income changes which the new export of A may bring about. The autonomous increase in exports of country A, purchased by people in country B who substitute imported for domestic products, is likely to cause an income decline in country B; and a subsequent decline of B's imports would, of course, work back upon the *net* export volume of A.

In simplest language the explanation of the foreign repercussion is this. A's export is B's import; if A's export raises incomes in A, it will, just as likely, reduce incomes in B; just as the income growth in A will induce increased imports of A, the income decline in B will induce decreased imports of B; but B's imports are, in part at least, A's exports; and this "foreign-induced" decline in A's exports must be taken into consideration.

For Model III we assume that there are no other countries besides A and B; hence, the induced import decline of B must be at the same time and in its entirety an export decline of A. (Cases with more than two countries will be dealt with later on.) That all amounts are expressed in dollars for both countries concerned is justifiable in view of the assumption of

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fixed foreign-exchange rates.[1] The terminological innovation of speaking of "foreign-induced exports" and "foreign-induced decline in exports," in contradistinction to "home-induced imports" and "home-induced decline in imports," is, I hope, self-explanatory: Foreign-induced changes in trade are those attributable to income changes in foreign countries, home-induced changes are those attributable to changes in the home-country's national income.

We assume, for Model III, that the marginal propensity to import is $\frac{3}{10}$ in country A and only $\frac{2}{10}$ in country B. The income-propagation periods, i.e., the length of time a round of spending takes, are assumed to be the same in the two countries.

In Model III we again rule out leakages (seepages) through induced increases (decreases) in *saving*, doing this merely for the purpose of isolating the leakages (seepages) through induced increases (decreases) in *imports*. That is to say, adjustments in foreign trade, home-induced and foreign-induced, will be seen in solo-performance, instead of sharing the show with adjustments in saving. The assumptions which may explain the absence of induced changes in saving are the same as for Model I above.

Table III follows the series of successive spendings and successive failures to spend in both countries up to the eighth period, that is through a time interval of something like 21–24 months. For each of the two countries, from the second period on, the changes[2] in foreign trade are shown in three items.

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[1] The same procedure and justification is used by Lloyd A. Metzler, "The transfer problem reconsidered," *Journal of Political Economy*, Vol. 50, 1942, p. 398.

[2] It may be well to repeat that all increases and decreases referred to here and anywhere in this analysis are differences in comparison with the magnitudes of the initial position—not changes in comparison with the immediately preceding period.

For country A there is

(*a*) the autonomous increase in exports;

(*b*) the foreign-induced decrease in exports (induced by the income decline of B); and

(*c*) the home-induced increase in imports (induced by the income increase of A).

For country B there is

(*d*) the autonomous increase in imports;

(*e*) the foreign-induced increase in exports (induced by the income increase of A); and

(*f*) the home-induced decrease in imports (induced by the income decline of B).

It goes without saying that (*a*) must be equal in magnitude to (*d*), (*b*) to (*f*), and (*c*) to (*e*).[1]

Let us follow the events in country B through a few rounds of spending (or, rather, non-spending). In the first period, the green cheese import wipes out $100.00 of B income when people prefer to substitute the foreign product for home-made products. The primary sufferers of the income decline will, in the second period, reduce their expenditures; but not all of these omitted expenditures concern other nationals of B; two tenths are lost to foreigners and only eight tenths constitute a loss of income to producers in B. The decline of income which, in this second period, nationals of B suffer, would thus amount to $80.00 in addition to the $100.00 which are lost to the producers who continue to be outrivaled by the foreign cheese; that would be altogether $180.00, were it not for the foreign-induced increase in exports, in an amount of $30.00, which causes the net loss of income to be only $150.00. In the third period, 80 per cent of this income loss is propagated to

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[1] Absolute equality of one country's exports with the other countrys' imports must hold, at least, for the case of two countries because there is no third country collecting for transportation of the goods passing through it in transit. We shall assume this equality also for the case of more countries.

other groups of people; this accounts for a "secondary" income decline of $120.00; add to this the "primary" $100.00 which the import-victims again fail to earn; but then deduct from these deficits, aggregating $220.00, the gain of $45.00 which comes to the producers of foreign-induced exports; and a net-deficit of $175.00 is seen to result for the period.

	COUNTRY A: S=0.0 M=0.3								COUNTRY B: S=0.0 M=0.2							
Period	K=2			Foreign Balance				Income from 2+5+6	K=2			Foreign Balance				Income from 10+13-14
	Spendable Income			=12	=14	=-15			Spendable Income			=4	=6	=-7		
	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For.-Ind ΔX	Auton. ΔX	Net Δ(X-M)	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For.-Ind ΔX	Auton. ΔM	Net Δ(X-M)	Current ΔY
1						100.00	100.00	100.00						+100.00	-100.00	-100.00
2	100.00	70.00		30.00	-20.00	100.00	50.00	150.00	-100.00	-80.00		-20.00	+30.00	+100.00	-50.00	-150.00
3	150.00	105.00		45.00	-30.00	100.00	25.00	175.00	-150.00	-120.00		-30.00	+45.00	+100.00	-25.00	-175.00
4	175.00	122.50		52.50	-35.00	100.00	12.50	187.50	-175.00	-140.00		-35.00	+52.50	+100.00	-12.50	-187.50
5	187.50	131.25		56.25	-37.50	100.00	6.25	193.75	-187.50	-150.00		-37.50	+56.25	+100.00	-6.25	-193.75
6	193.75	135.63		58.12	-38.75	100.00	3.13	196.88	-193.75	-155.00		-38.75	+58.12	+100.00	-3.13	-196.88
7	196.88	137.82		59.06	-39.38	100.00	1.56	198.44	-196.88	-157.50		-39.38	+59.06	+100.00	-1.56	-198.44
8	198.44	138.91		59.53	-39.69	100.00	.78	199.22	-198.44	-158.75		-39.69	+59.53	+100.00	-.78	-199.22
∞	200.00	140.00		60.00	-40.00	100.00	0	200.00	-200.00	-160.00		-40.00	+60.00	+100.00	0	-200.00

TABLE III.—Autonomous export from A to B. Induced imports and foreign repercussions. No saving.

The income decline in *B* and the income growth in *A* can be seen to move here absolutely in step. The reason for the equal pace is that the leakage in *A* and the seepage in *B* are of equal size. To be sure, the marginal propensities to import are not equal; the home-induced increases in the imports of *A* are three tenths of the income increments in *A*, and the home-induced decreases in the imports of *B* are only two tenths of the income decrements in *B*. But the foreign repercussions are balancing the two forces. The foreign-induced increase in the exports of *B* adds to the seepage in *B*, and the foreign-induced fall in the exports of *A* adds to the leakage in *A*, precisely the amounts which result from each other's propensities to import. The absolute sums of home-induced and foreign-induced

changes in foreign trade are, therefore, equal in the two countries.   It should be noted, however, that this is so only as long as no other unequal forces, such as unequal propensities to save, affect the income adjustments in the two countries.

The final income increase is $200.00 in A, and the final income decline is $200.00 in B.   Thus, the multiplier is 2 in both countries.   It is worth emphasizing that the propensities to consume domestic goods and services are different in A and B, namely $7/10$ in A and $8/10$ in B.   Nevertheless, the multipliers (which without foreign repercussions would be $3.33\frac{1}{3}$ in A and 5 in B) are here, because of the foreign repercussions, 2 in A and also 2 in B.

Using provisionally (reserving justification for later) the formula $k = \dfrac{1}{s + m + f}$ for this case, where $s = 0$ in both countries, $m_A = .3$, $m_B = .2$, $f_A = m_B$, and $f_B = m_A$, we can see that the one multiplier, $k_A = \dfrac{1}{0 + .3 + .2} = 2$, and also the other multiplier, $k_B = \dfrac{1}{0 + .2 + .3} = 2$.

SELF-ADJUSTMENT OF THE FOREIGN BALANCE

In Model III the trade balance rectifies itself completely. The export surplus of A declines rapidly from the $100.00 resulting from the uncompensated autonomous export in the beginning.   The induced reduction of other exports of A together with the induced increase in the imports of A cut A's export surplus by 50 per cent each period.   In the second period the export surplus of A (and import surplus of B) is only $50.00, and in the fifth period it is no larger than $6.25.   The rate at which the foreign balance vanishes is, of course, determined by the sum of the home country's and the foreign country's propensities to import, which is five tenths in the present case.   Each export (import) surplus produces income

in A (destroys income in B), but half of this increment (decrement) in income becomes a foreign-trade leakage (seepage) in the subsequent period, reducing the next export (import) surplus by 50 per cent.

Enlightened by the analysis of Model II, we know why the self-adjustment of the trade balance, not accomplished there, is achieved here in Model III. The absence of induced saving in Model III does it. If no leakage through saving helps in checking the rise of income in A, the foreign factors—home-induced imports and foreign-induced decreases in exports—must do the whole job. Hence, if these induced changes in the trade balance are to bring the rise in income to a stop, they must become equal to the autonomous change in the trade balance. Income, in this case, rises until the trade balance is rectified, whereas in other cases, with induced saving working in unison with the other factors, the rise in incomes is estopped before the trade balance is rectified.

The questions may now be asked whether the self-adjustment of the trade balance (via income effects only) leaves the two countries without any changes in their foreign indebtedness and without any changes in their domestic money circulations. Both questions must be answered in the negative. The foreign balances during the adjustment period are not offset, then or later, by opposite balances; in other words, there is no reversal of the direction of flows of goods and funds but merely a gradual disappearance. The sum of foreign balances accumulated for all periods, beginning from the appearance of the export surplus until its disappearance, is, in the present case, $200.00. This amount is equal to that of the final income increase in A (and decrease in B). These $200.00 may best be visualized as gold or foreign bank balances acquired by the monetary authority of A, which thereby creates A-money in the same amount. The newly created $200.00 in A-money circulate from then on and produce

$200.00 income each period.   (The same result can be visual-
ized with newly activated, rather than newly created funds.)
The monetary circulation and the income level are increased
in *A* (and decreased in *B*) "for good," that is to say, for as long
as the cheese exports continue and no other autonomous
changes occur.

IMPORT RESTRICTION VERSUS EXPORT INCREASE

We mentioned at the outset of our demonstration of the
modus operandi of the foreign-trade multiplier that the case of an
autonomous import reduction, under the chosen set of assump-
tions, would be fully analogous to that of the autonomous
export increase, which is the standard case of this analysis.
That an export increase and an import reduction, thrown
together under the heading of "improvements of the trade
balance," are equivalent in most of their effects is often an
absolutely unwarranted contention; but under the strict
regime of the chosen assumptions, the equivalence holds.

In the case of an additional export the producers of the
export article are the recipients of the primary new income; in
the case of an omitted import the producers of the home-made
goods which are substituted for the imported ones are the
recipients of the primary new income.   The new money
which enters into the income flow originates in both cases from
creation or activation of funds upon acquisition of gold,
foreign exchange or other assets, according to the explana-
tions given in an earlier section.[1]

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[1] See above p. 14 ff.   One might object that we speak of "new money"
when in fact money merely fails to leak out of the income flow in payment
for foreign products.   Yet, starting from an equilibrium position, the reduc-
tion of imports with a temporarily maintained volume of exports does con-
stitute a net supply of gold, foreign exchange or other assets to banks or
holders of liquid funds and, thus, does involve a release of "new money"
into circulation.

In the case of the autonomous reduction of imports in country A, the generated incomes, primary and secondary, will engender induced increases of imports in amounts determined by A's marginal propensity to import. Country B, however, which through the autonomous change has lost exports and suffered reductions in income, primary as well as secondary, will reduce its purchases from abroad to an extent determined by its marginal propensity to import. This constitutes an induced reduction in exports of A. Eventual equilibrium will find incomes increased in A and reduced in B, the respective increase and reduction sufficient to make the induced import increase and the induced export decrease of A equal to its autonomous import reduction. In cases where induced saving is involved—as in all our models except I and III— the incomes will change by different degrees (and the trade balances will not be rectified), but the principle is the same: the import reduction of A leads to a permanently increased level of its income.[1]

This contention concerning a "permanently" increased level of national income due to a favorable trade balance, may be said to characterize the discussed theory as "neo-mercantilistic." However, more properly, only those economists should be called neo-mercantilists who have such a high confidence in the unrestricted applicability of the theory that they would be willing to endorse protectionist commercial policies. The present writer is too much impressed with the artificiality of the assumptions to accept the conclusions as a sufficient basis for policy recommendations. After all, if I may be permitted to reiterate, the assumptions include unchanged commodity prices, unchanged interest rates,

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[1] Compare this with the opposite contention of P. T. Ellsworth, *op. cit.*, p. 334: "For a period of time determined by the speed with which exports fall off from their initial sustained level, employment and incomes are increased by an amount which is at first considerable, but which gradually tapers off to nothing."

unchanged foreign-exchange rates and, of course, absence
of further "autonomous" changes in trade such as would be
implied in retaliatory import restrictions by the adversely
affected country.   This is just too much to neglect.

But for purposes of a clean analysis we stick to our assump-
tions.

INVISIBLE EXPORTS AND IMPORTS

Our exposition so far has been in terms of exports, imports
and trade balances, and, thus, pointed to the merchandise
items in the international balance of payments.   Some
remarks concerning the "service items" in the balance of
payments are now in order.   (The capital items in the
balance of payments will be dealt with in greater detail in
Chapter VIII.)

Service items are customarily listed together with the mer-
chandise items as "transactions on current account" (or
"income account").   The easiest way to include them in our
analysis would be to state that whenever we speak of exports
or imports we mean these words to comprise "invisible"
exports or imports, respectively.   Services rendered to for-
eigners, as well as gifts, dividend and interest payments
received from abroad, would be counted among exports;
services received from foreigners, as well as gifts, dividend and
interest payments made to foreign countries would be counted
among imports.

This solution would indeed be simple; but, unfortunately,
it would be grossly misleading, at least as far as the receipt
of interest and dividend payments is concerned.   The hitch is
in the drastic difference of the marginal propensities of capital
income recipients from the "over-all" marginal propensities
to use national income.[1]   The income from additional
merchandise exports is received in the form of wages, rents

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[1] For this point I am indebted to Professor Harold M. Somers.

and profits, without constituting a significant change in the distribution of the national income. The same can probably be said of the income received from additional sales of certain services to foreigners, such as shipping or tourist services. But the additional receipt of foreign dividend payments is a different matter. The dividend recipients' marginal propensity to consume is likely to be so much smaller than that of the average income recipient, that the assumption of stable marginal propensities is definitely incompatible with that change in income distribution which is involved in a change in dividend receipts from abroad.

Hence, a more complete theory must take account of these large differences in the propensities of different income recipients. The "foreign-dividend multiplier" will be smaller than the foreign-trade multiplier. Nevertheless, for a discussion of other matters pertinent to invisible exports and imports, we permit the items for capital service to stay under the same heading as the other service items.

Changes in any of these items, just as in merchandise items, may be regarded as induced or autonomous according as to whether or not they were consequential upon changes in the national income of the country purchasing the services or making the payments in question. Induced changes in invisible imports are probably less significant than induced changes in merchandise imports, both in absolute amounts and in proportion to the total magnitudes of the respective items. Nevertheless, the income effects upon invisible imports need not be negligible. Amounts spent on foreign travel, remittances to foreign relatives, dividend payments to foreign share-holders may all show considerable income-elasticity,[1]

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[1] Besides income-elasticity these items are also subject to a certain price-elasticity. For a discussion of the price-elasticity (and exchange-rate-elasticity) of invisible imports and exports see my article "The theory of foreign exchanges," *Economica*, Vol. 6 (New Series), 1939, pp. 391–2.

that is to say, they may all vary fairly conspicuously as functions of national income.

Autonomous changes in invisible exports as well as imports occur probably all the time, if only by relatively minor amounts. However, sudden autonomous changes of considerable magnitude may take place, for example, if foreign exchange restrictions are applied in any of the countries concerned. A transfer moratorium of a debtor nation stopping all payments of dividends and interest to foreigners, will be analogous in the no-longer-paying country to an autonomous reduction of imports, and in the no-longer-receiving country to an autonomous reduction of exports—subject, of course, to the previously mentioned qualifications concerning the probably different multiplier effects of changed capital incomes from those of changes in other incomes.

Considerable autonomous changes in tourist traffic may take place over a relatively short time, owing to changing fashions (winter sport), political climates (danger of war or revolution, economic warfare[1]), transportation factors (opening or closing of railroad or shipping lines[2]), propaganda campaigns ("Come to Canada," "Know your country first") or the introduction of special measures (cheaper exchange rates for tourists).

We refer only in passing to the statistical difficulties of dealing with the autonomous and induced changes in service items. These difficulties are in principle as bad as the statistical difficulties connected with merchandise items. They may be even worse, because the basic statistical data on invisible items are much less complete than those on visible

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[1] Nazi Germany succeeded in throttling the Austrian tourist business during the years preceding the annexation in 1938. The fifth-column-tourists were not perfect substitutes for real vacationists.

[2] The U-boat warfare killed the tourist business of the Bahama Islands in 1942.

ones.　But they are probably less serious, because the amounts involved are usually much smaller than the merchandise trade.[1]

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[1] The correctness of including invisible items on current account in exports and imports is almost self-evident to most writers on multiplier theory. See, e.g., Colin Clark and J. G. Crawford, *The National Income of Australia*, p. 93, who state that "a decline in the annual amounts payable overseas on account of interest and dividends has exactly the same effect as an increase in export income, and should be included with it." Apart from the possible differences in propensities and, thus, in multiplier effects, the statement neglects to discriminate between different causes of the changes in export income and dividend payments. The decline in dividend payments may be induced by a decline in domestic income, or the increase in exports may be induced by an increase in foreign income. An induced export increase, resulting from an *increase* of incomes abroad, is certainly not analogous in its effects to a decline in dividend payments to foreigners, which results in a *decrease* of incomes abroad and, therefore, may have repercussions on the volume of exports.

Chapter V

THE FOURTH MODEL: INDUCED IMPORTS, EXPORTS AND SAVING. THE MULTIPLIER FORMULA INVOLVING TWO COUNTRIES. THE SIZE OF THE FOREIGN BALANCE. THE REVISED MULTIPLIER AS A FUNCTION OF TIME

MODEL IV: INDUCED SAVING, FOREIGN REPERCUSSIONS

We can now proceed to have induced savings, induced imports and foreign repercussions perform in ensemble. This is done in Model IV, which, however, is still confined to two countries only. Thus, we still have the convenient condition that A's export surplus is identical with B's import surplus, and we have to deal with only four variables, namely s_A, m_A, s_B, and m_B, that is, the marginal propensities to save and to import in A and in B.

The sequences and their results vary so much with different magnitudes of, and relationships between, the propensities to save and import in the two countries, that it will be worth our while to go through a number of tables, each based on a different set of propensities in country B with an unchanged set of propensities in A. Let us assume that in A the marginal propensity to save is .2 and the marginal propensity to import is .3. For country B we shall start with the assumption of propensities of half the respective magnitudes of A, that is, $s_B = .1$ and $m_B = .15$.

Table IV-a is worked out on the basis of these assumptions. The table follows the series of successive spendings and successive failures to spend in the two countries up to the 13th period, that is, through a time interval of something like 36–39 months. Explanations of the columns were given above when Table III was exhibited. But, in order to familiarize ourselves with the procedure, we may follow here

	COUNTRY A. S=0.2 M=0.3								COUNTRY B: S=0.1 M=0.15							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	K=1.25			Foreign Balance				Income from 2+5+6	K=2.50			Foreign Balance				Income from 10+13-14
	Spendable Income			=12	=14	=-15			Spendable Income			=4	=6	=-7		
Period	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Auton. ΔX	Net Δ(X-M)	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Auton. ΔM	Net Δ(X-M)	Current ΔY
1						100.00	100.00	100.00						+100.00	-100.00	-100.00
2	100.00	50.00	20.00	30.00	-15.00	100.00	55.00	135.00	-100.00	-75.00	-10.00	-15.00	+30.00	+100.00	-55.00	-145.00
3	135.00	67.50	27.00	40.50	-21.75	100.00	37.75	145.75	-145.00	-108.75	-14.50	-21.75	+40.50	+100.00	-37.75	-168.25
4	145.75	72.87	29.15	43.73	-25.24	100.00	31.03	147.63	-168.25	-126.19	-16.82	-25.24	+43.73	+100.00	-31.03	-182.46
5	147.63	73.81	29.53	44.29	-27.37	100.00	28.34	146.44	-182.46	-136.84	-18.25	-27.37	+44.29	+100.00	-28.34	-192.55
6	146.44	73.22	29.29	43.93	-28.88	100.00	27.19	144.34	-192.55	-144.41	-19.26	-28.88	+43.93	+100.00	-27.19	-200.48
7	144.34	72.17	28.87	43.30	-30.07	100.00	26.63	142.10	-200.48	-150.36	-20.05	-30.07	+44.30	+100.00	-26.63	-207.06
8	142.10	71.05	28.42	42.63	-31.06	100.00	26.31	139.99	-207.06	-155.30	-20.70	-31.06	+42.63	+100.00	-26.31	-212.67
9	139.99	69.99	28.00	42.00	-31.90	100.00	26.10	138.09	-212.67	-159.50	-21.27	-31.90	+42.00	+100.00	-26.10	-217.50
10	138.09	69.04	27.62	41.43	-32.62	100.00	25.95	136.42	-217.50	-163.13	-21.45	-32.62	+41.43	+100.00	-25.95	-221.70
11	136.42	68.21	27.28	40.93	-33.25	100.00	25.82	134.96	-221.70	-166.28	-22.17	-33.25	+40.93	+100.00	-25.82	-225.35
12	134.96	67.48	26.99	40.49	-33.80	100.00	25.71	133.68	-225.35	-169.01	-22.54	-33.80	+40.49	+100.00	-25.71	-228.52
13	133.68	66.84	26.74	40.10	-34.28	100.00	25.62	132.56	-228.52	-171.39	-22.85	-34.28	+40.10	+100.00	-25.62	-231.29
∞	125.00	62.50	25.00	37.50	-37.50	100.00	25.00	125.00	-250.00	-187.50	-25.00	-37.50	+37.50	+100.00	-25.00	-250.00

TABLE IV-a.—Autonomous export from A to B. Induced saving, imports and foreign repercussions.

the events in country A, as recorded in Table IV-a, through a few rounds of spending.

In the first period, the new export (autonomous ΔX) creates a new income (current ΔY) of $100. Acting according to their propensities, the recipients of this income will, in the second period, spend $50 on home-produced consumption goods (current ΔC), $30.00 on imported goods (home-induced ΔM) and will save $20.00 (induced ΔS). The contributions to the income of this period—$100 from a repetition of the export

order and $50 from the increased expenditures for home consumption—will be abated by a reduction in exports in an amount of $15 (foreign-induced $- \Delta X$), a reduction which results from the income decline in the foreign country. Thus, the net contribution to the income of the second period is $135. This amount is distributed in the third period, again in the assumed proportions, among home consumption ($67.50), import ($40.50) and saving ($27.00); and the income of the third period gets, on the one hand, the boost from the consumer's expenditures ($67.50) and the repeated export order ($100.00), and, on the other hand, the cut from lost exports of other goods ($21.75), lost because of the income reduction abroad. The net effect upon income in A is, in this period, $145.75.

Observing, in Table IV-a, the whole sequence of incomes in A and B, we find that the income decline in B is greater than the income increase in A. This conforms to what probably most of us would have expected in view of the fact that the propensity to save is smaller in B than in A, so that the seepages through decreasing saving would do less toward arresting B's income decline than the leakages through increased saving would do toward arresting A's income increase. The final income increase in A is $125.00, the final income decrease in B $250.00, which, with the autonomous export of $100, makes the multipliers $k_A = 1.25$ and $k_B = 2.50$. What is surprising, probably to most of us, is that the income increase in A exhibits a curious wave, rising first up to $147.63 in the fourth period and then slowly falling towards the final $125.00. The cause of this wave will have to be investigated later. Figure 2 pictures the time series of incomes and trade balances.

If one looked at A's marginal propensity to save of .2 and neglected everything else, he would expect to find a multiplier of five. Taking account also of A's marginal propensity to import of .3, but still neglecting the foreign repercussions, a
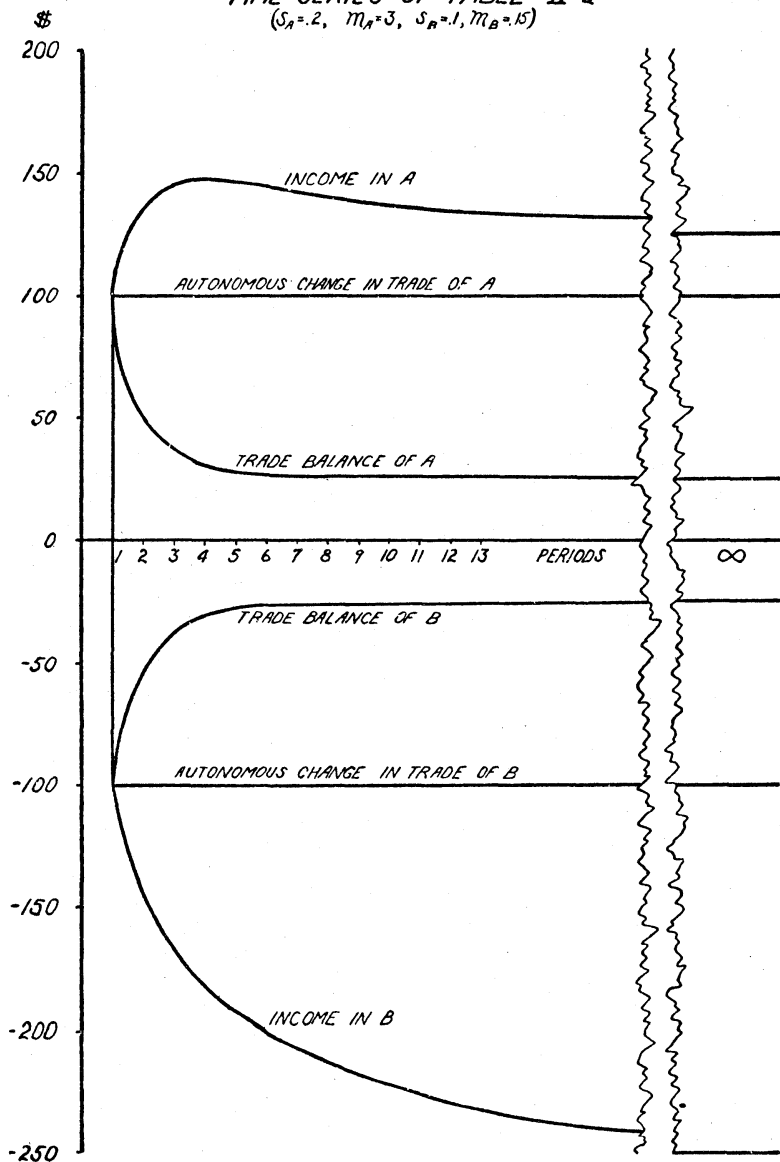
FIG. 2.—Time series of Table IV-a.

multiplier of two would be expected. Now including in one's considerations the foreign repercussions, he finds that this depresses the multiplier in A to one and one-fourth. The first object of our curiosity is now to see what will be the effects upon the foreign repercussions and the multiplier in A if the propensities to save and to import are twice as high in B as we have first assumed. Thus, let us make $s_B = .2$ and $m_B = .3$, which, incidentally, makes them equal to the respective values in A.

	COUNTRY A : s=0.2 M=0.3							COUNTRY B : s=0.2 M=0.3							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	K=1.25			Foreign Balance			Income from 2+5+6	K=1.25			Foreign Balance			Income from 10+13+14
	Spendable Income			=12	=14	=-15			Spendable Income			=4	=6	=-7	
Period	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Auton. ΔX	Net Δ(X·M)	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Auton. ΔM	Net Δ(X·M)	Current ΔY
1						100.00	100.00	100.00						+100.00	-100.00	-100.00
2	100.00	50.00	20.00	30.00	-30.00	100.00	40.00	120.00	-100.00	-50.00	-20.00	-30.00	+30.00	+100.00	-40.00	-120.00
3	120.00	60.00	24.00	36.00	-36.00	100.00	28.00	124.00	-120.00	-60.00	-24.00	-36.00	+36.00	+100.00	-28.00	-124.00
4	124.00	62.00	24.80	37.20	-37.20	100.00	25.60	124.80	-124.00	-62.00	-24.80	-37.20	+37.20	+100.00	-25.60	-124.80
5	124.80	62.40	24.96	37.44	-37.44	100.00	25.12	124.96	-124.80	-62.40	-24.96	-37.44	+37.44	+100.00	-25.12	-124.96
6	124.96	62.48	24.99	37.49	-37.49	100.00	25.02	124.99	-124.96	-62.48	-24.99	-37.49	+37.49	+100.00	-25.02	-124.99
∞	125.00	62.50	25.00	37.50	-37.50	100.00	25.00	125.00	-125.00	-62.50	-25.00	-37.50	+37.50	+100.00	-25.00	-125.00

TABLE IV-b.—Autonomous export from A to B. Induced saving, imports and foreign repercussions.

The sequence and end effects arising from the changed assumptions are shown in Table IV-b. We can see that the doubling of the propensities in B has cut the multiplier in B to half its former value; but the foreign repercussion and the multiplier in A are not affected at all. The doubling of the foreign propensities does not change the foreign repercussion on A. The multiplier in A is still 1.25, and the multiplier in B is now equal to it (as was to be expected with an equal set of propensities). The sequences of incomes proceed this time at an identical pace in the two countries, with no wavelike movement, and, indeed, at a very fast pace: the income change

of the second period is already within ten per cent of the final level.

Let us try another variation: this time we leave the marginal propensity to save in B unchanged, thus $s_B = .2$, but reduce B's marginal propensity to import from .3 to .2. The propensities to save are now equal in A and B, while the propensity to import is lower in B than in A.

The consequences are shown in Table IV-c. The income changes proceed without waves and at an equal pace in the

	COUNTRY A: S=0.2 M=0.3								COUNTRY B: S=0.2 M=0.2							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	K=1.429	Foreign Balance						Income from	K=1.429	Foreign Balance						Income from
	Spendable Income					=12	=14	=-15	Spendable Income					=4	=6	=-7
Period								2+5+6								10+13-14
	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Auton. ΔX	Net Δ(X-M)	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Auton. ΔM	Net Δ(X-M)	Current ΔY
1						100.00	100.00	100.00						+100.00	-100.00	-100.00
2	100.00	50.00	20.00	30.00	-20.00	100.00	50.00	130.00	-100.00	-60.00	-20.00	-20.00	+30.00	+100.00	-50.00	-130.00
3	130.00	65.00	26.00	39.00	-26.00	100.00	35.00	139.00	-130.00	-78.00	-26.00	-26.00	+39.00	+100.00	-35.00	-139.00
4	139.00	69.50	27.80	41.70	-27.80	100.00	30.50	141.70	-139.00	-83.40	-27.80	-27.80	+41.70	+100.00	-30.50	-141.70
5	141.70	70.85	28.34	42.51	-28.34	100.00	29.15	142.51	-141.70	-85.02	-28.34	-28.34	+42.51	+100.00	-29.15	-142.51
6	142.51	71.26	28.50	42.75	-28.50	100.00	28.75	142.76	-142.51	-85.51	-28.50	-28.50	+42.75	+100.00	-28.75	-142.76
7	142.76	71.38	28.55	42.83	-28.55	100.00	28.63	142.83	-142.76	-85.66	-28.55	-28.55	+42.83	+100.00	-28.63	-142.83
∞	142.86	71.43	28.57	42.86	-28.57	100.00	28.57	142.86	-142.86	-85.72	-28.57	-28.57	+42.86	+100.00	-28.57	-142.86

TABLE IV-c.—Autonomous export from A to B. Induced saving, imports and foreign repercussions.

two countries; the final income changes in A and in B are identical in size, namely $142.86; thus the multipliers in both countries are 1.429, which is higher than in the former case when B's propensity to import was higher. We are tempted to make the generalizations that the multipliers in the two countries will equal each other if the propensities to save are equal, even if the propensities to import are different; and, furthermore, that variations in propensities to import will affect the size of both multipliers but will not interfere with their equality. The truth of these generalizations will be

proved later when we analyze the properties of the "foreign repercussion" term in the multiplier formula.

MORE VARIATIONS OF MODEL IV

In the last case s_B was .2, and m_B was likewise .2 (while the propensities in country A, as in all cases of this Model IV, were $s_A = .2$ and $m_A = .3$). Let us now reduce s_B to .1 and raise m_B to .45.

	COUNTRY A : S=0.2 M=0.3								COUNTRY B : S=0.1 M=0.45							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	K= .714			Foreign Balance				Income from 2+5+6	K=1.429			Foreign Balance				Income from 10+13-14
Period	Spendable Income			=12	=14	=-15			Spendable Income			=4	=6	=-7		
	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Auton. ΔX	Net Δ(X-M)	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Auton. ΔM	Net Δ(X-M)	Current ΔY
1						100.00	100.00	100.00						+100.00	-100.00	-100.00
2	100.00	50.00	20.00	30.00	-45.00	100.00	25.00	105.00	-100.00	-45.00	-10.00	-45.00	+30.00	+100.00	-25.00	-115.00
3	105.00	52.50	21.00	31.50	-51.75	100.00	16.75	100.75	-115.00	-51.75	-11.50	-51.75	+31.50	+100.00	-16.75	-120.25
4	100.75	50.37	20.15	30.23	-54.11	100.00	15.66	96.26	-120.25	-54.11	-12.03	-54.11	+30.23	+100.00	-15.66	-123.88
5	96.26	48.13	19.25	28.88	-55.74	100.00	15.38	92.39	-123.88	-55.74	-12.39	-55.74	+28.88	+100.00	-15.38	-126.87
6	92.39	46.19	18.48	27.71	-57.09	100.00	15.20	89.11	-126.87	-57.09	-12.69	-57.09	+27.71	+100.00	-15.20	-129.38
7	89.11	44.56	17.83	26.73	-58.22	100.00	15.05	86.33	-129.38	-58.22	-12.94	-58.22	+26.73	+100.00	-15.05	-131.49
8	86.33	43.17	17.27	25.90	-59.17	100.00	14.93	83.99	-131.49	-59.17	-13.15	-59.17	+25.90	+100.00	-14.93	-133.27
9	83.99	42.00	16.80	25.20	-59.97	100.00	14.83	82.02	-133.27	-59.97	-13.33	-59.97	+25.20	+100.00	-14.83	-134.77
10	82.02	41.01	16.40	24.61	-60.65	100.00	14.74	80.36	-134.77	-60.65	-13.48	-60.65	+24.61	+100.00	-14.74	-136.03
11	80.36	40.18	16.07	24.11	-61.21	100.00	14.68	78.97	-136.03	-61.21	-13.60	-61.21	+24.11	+100.00	-14.68	-137.11
∞	71.43	35.71	14.29	21.43	-64.29	100.00	14.28	71.43	-142.86	-64.29	-14.28	-64.29	+21.43	+100.00	-14.28	-142.86

TABLE IV-d.—Autonomous export from A to B. Induced saving, imports and foreign repercussions.

In Table IV-d we see what happens. The multiplier in country B is the same as before, namely, 1.429. B's income decline proceeds, however, at a much slower pace than before. In the former case the income decline was in the second period already within 10 per cent of the final level; in this case it takes six periods to get that far and the further approach to the full multiple is remarkably slow. In country A the income increase exhibits now again a wavelike movement. The highest income occurs in the second period, when the income

TIME SERIES OF TABLE IV-d
$(s_A = .2, \ m_A = .3, \ s_B = .1, \ m_B = .45)$
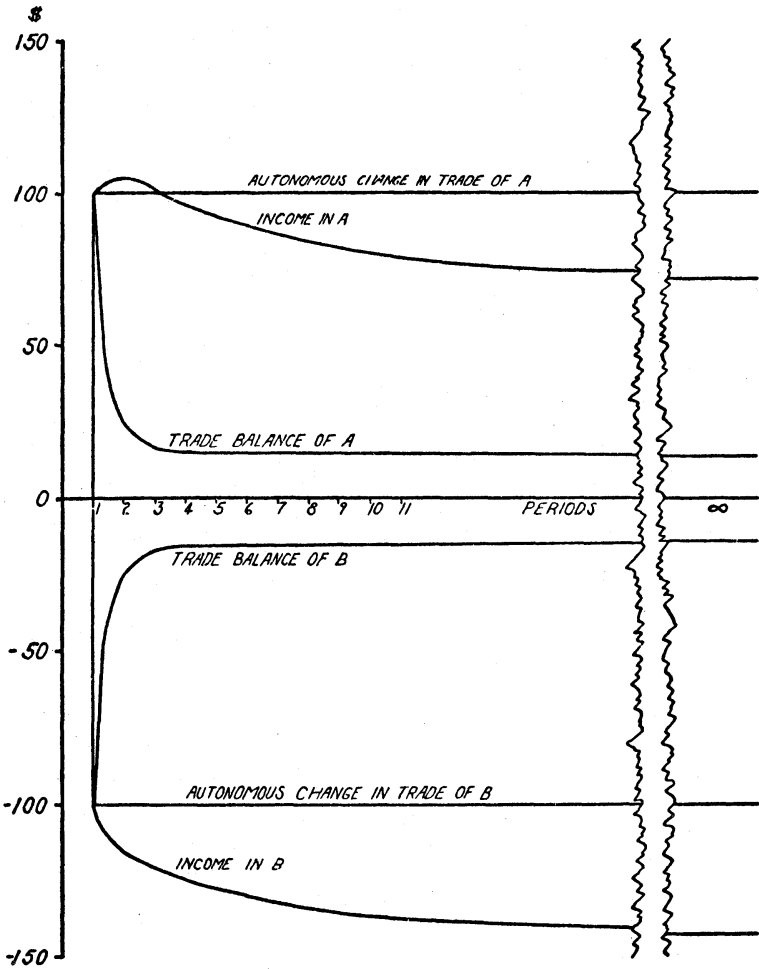

FIG. 3.—Time series of Table IV-d.

increase amounts to $105.00; from then on income slowly recedes; it takes eleven periods to get within 10 per cent of its final value; and this final income increase is only $71.43. This means, with the autonomous export amounting to $100, that the multiplier is less than one, or .714.

This multiplier, it will be observed, is exactly half the multiplier in country B. The main fact which this case teaches us is, however, that *the foreign repercussion can be powerful enough to reduce the multiplier below one*. The multiplier would

	COUNTRY A · S=0.2 M=0.3								COUNTRY B : S=0.45 M=0.1							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	K=1.837			Foreign Balance				Income from 2+5+6	K=.816			Foreign Balance				Income from 10+13-14
	Spendable Income			=12	=14	=-15			Spendable Income			=4	=6	=-7		
Period	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Auton ΔX	Net Δ(X·M)	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Auton ΔM	Net Δ(X·M)	Current ΔY
1						100.00	100.00	100.00						+100.00	-100.00	-100.00
2	100.00	50.00	20.00	30.00	-10.00	100.00	60.00	140.00	-100.00	-45.00	-45.00	-10.00	+30.00	+100.00	-60.00	-115.00
3	140.00	70.00	28.00	42.00	-11.50	100.00	46.50	158.50	-115.00	-51.75	-51.75	-11.50	+42.00	+100.00	-46.50	-109.75
4	158.50	79.25	31.70	47.55	-10.97	100.00	41.48	168.28	-109.75	-49.39	-49.39	-10.97	+47.55	+100.00	-41.48	-101.84
5	168.28	84.14	33.66	50.48	-10.18	100.00	39.34	173.96	-101.84	-45.83	-45.83	-10.18	+50.48	+100.00	-39.34	-95.34
6	173.96	86.98	34.79	52.19	-9.53	100.00	38.28	177.45	-95.35	-42.91	-42.91	-9.53	+52.19	+100.00	-38.28	-90.72
7	177.45	88.73	35.49	53.23	-9.07	100.00	37.70	179.66	-90.72	-40.82	-40.82	-9.07	+53.23	+100.00	-37.70	-87.59
∞	183.67	91.84	36.73	55.10	-8.16	100.00	36.73	183.67	-81.63	-36.73	-36.73	-8.16	+55.10	+100.00	-36.73	-81.63

TABLE IV-e.—Autonomous export from A to B.   Induced saving, imports and foreign repercussions.

be five if merely the propensity to save were considered; the multiplier would be two if the propensity to import were also taken into account; but the multiplier is less than one if due account is taken of the foreign repercussion. The development of incomes and trade balances is pictured in Figure 3.

Continuing our variations, let us interchange the values for B's marginal propensities to save and to import. In the last case s_B was .1 and m_B was .45. Now let s_B be .45 and m_B be .1, thus leaving the sum of the propensities unchanged.

The effects are seen in Table IV-e. The multiplier in A jumps to 1.837, the multiplier in B falls to .816. The period
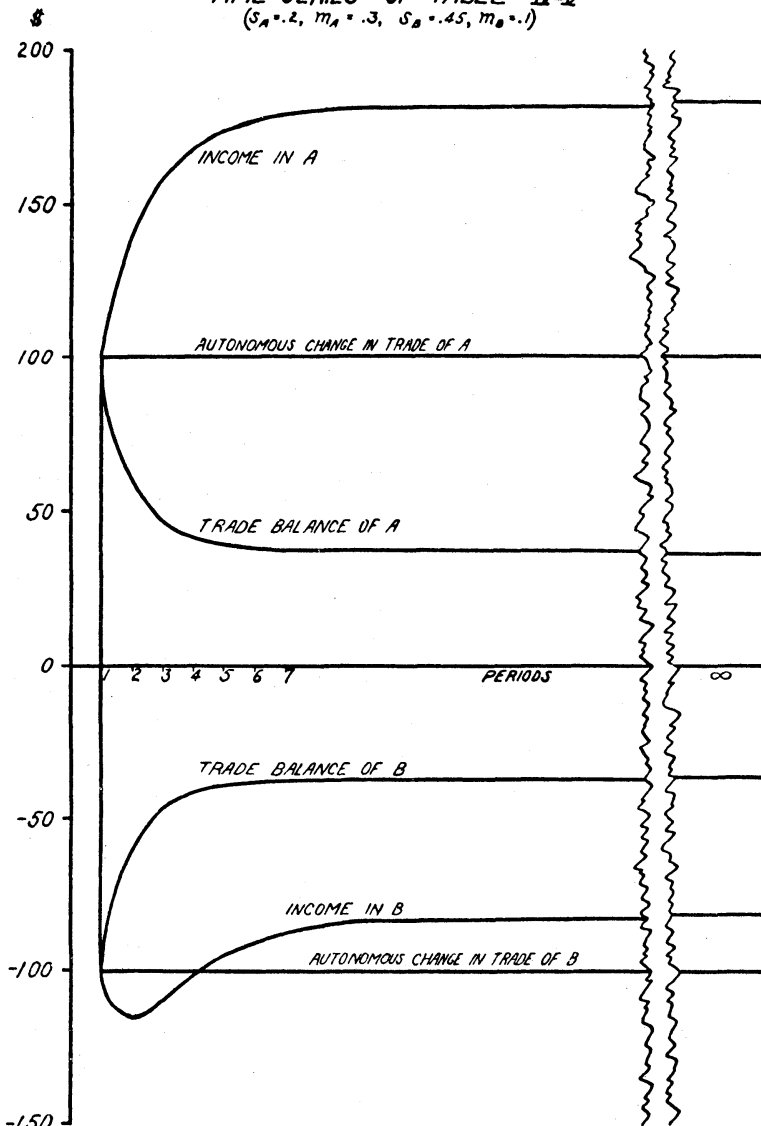
FIG. 4.—Time series of Table IV-e.

of adjustment is fairly short; in the fourth income propagation period the income increase in A is within 10 per cent of the full multiple. In country B the income decline moves in a wave; the lowest income is reached in the second period; from the fifth period on the income is reduced, as against the original level, by less than the autonomous import. It will be noted that in this case the multiplier in A has the highest value of all those observed in Model IV; in other words, the foreign repercussion is less forceful in this case than in any

	COUNTRY A : S=0.2 M=0.3								COUNTRY B : S=0.3 M=0.2							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	K=1.579				Foreign Balance			Income from 2+5+6	K=1.053				Foreign Balance			Income from 10+13+14
Period	Spendable Income				=12	=14	=-15		Spendable Income				=4	=6	=-7	
	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Auton. ΔX	Net Δ(X-M)	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Auton. ΔM	Net Δ(X-M)	Current ΔY
1						100.00	100.00	100.00					+100.00	-100.00	-100.00	-100.00
2	100.00	50.00	20.00	30.00	-20.00	100.00	50.00	130.00	-100.00	-50.00	-30.00	-20.00	+30.00	-100.00	-50.00	-120.00
3	130.00	65.00	26.00	39.00	-24.00	100.00	37.00	141.00	-120.00	-60.00	-36.00	-24.00	+39.00	-100.00	-37.00	-121.00
4	141.00	70.50	28.20	42.30	-24.20	100.00	33.50	146.30	-121.00	-60.50	-36.30	-24.20	+42.30	-100.00	-33.50	-118.20
5	146.30	73.15	29.26	43.89	-23.64	100.00	32.47	149.51	-118.20	-59.10	-35.46	-23.64	+43.89	-100.00	-32.47	-115.21
6	149.51	74.76	29.90	44.85	-23.04	100.00	32.11	151.72	-115.21	-57.61	-34.56	-23.04	+44.85	-100.00	-32.11	-112.76
7	151.72	75.86	30.34	45.52	-22.55	100.00	31.93	153.31	-112.76	-56.38	-33.83	-22.55	+45.52	-100.00	-31.93	-110.86
∞	157.89	78.94	31.58	47.37	-21.05	100.00	31.58	157.89	-105.26	-52.63	-31.58	-21.05	+47.37	-100.00	-31.58	-105.26

TABLE IV-f.—Autonomous export from A to B. Induced saving, imports and foreign repercussions.

other we have examined. A graphical representation of the movements of incomes and trade balances is given in Figure 4.

Let us inspect one more case of Model IV. We select s_B to be .3 and m_B to be .2. This case, worked out in Table IV-f, invites comparison with the cases shown in Tables IV-b and IV-c, the former based on $s_B = .2$ and $m_B = .3$, the latter on $s_B = .2$ and $m_B = .2$. In comparison with both these cases, the present one, with B's increased marginal propensity to save and reduced marginal propensity to import, respectively, shows a higher multiplier in A and a lower multiplier in B. And we find again a wave in the income sequence in B.

FURTHER OBSERVATIONS FROM TABLES IV-a–f

A few more observations about the sequences shown in the tables should be made before we turn from this, so-to-speak inductive study of our imaginary cases to their explanation or, that is, to deductive analysis.

A simple, almost self-evident observation is that waves in the sequences of savings and of induced imports occur whenever there are waves in the sequences of income changes. (And, of course, these waves will be felt in the other country, constituting there fluctuations in the sequences of the foreign-induced exports.) No waves were observed, however, in the sequences of net foreign balances. (Such waves will be found in Model V, when more than two countries are involved.) The export or import surpluses, starting always with the autonomous $100, were falling off steadily and fairly quickly.

Income in A increases as long as the autonomous exports exceed the induced savings, the induced imports and the foreign-induced fall in exports of the current period. Income in B decreases as long as the autonomous imports exceed the induced fall in savings, the induced fall in imports and the foreign-induced rise in exports of the current period. The current income can be calculated in several ways. The following three methods suggest themselves from inspection of the tables. First, the current income change (ΔY) is the algebraic sum of all changes in current disbursements to the nationals of the country, that is, of the consumption of home products (ΔC), the foreign-induced exports (For.-Ind. ΔX) and the autonomous exports (Auton. ΔX). Second, the current income change is the sum of the income change of the preceding period and the current foreign balance minus the induced savings of the period. Third, the current income change is the difference between the sums of all foreign

balances cumulated from the first period on and of all savings cumulated from the first period on:[1]

Clear from our previous discussions (especially in the section on "Induced Foreign Lending") is that the final level of induced saving must, since changes in home investment are ruled out, be equal to the final foreign balance. From this it will be clear why, in the ultimate equilibrium position, the ratio of the final net foreign balance per period to the final level of income must be equal to the marginal propensity to save.[2]

Finally, since the export surplus of A is always identical with the import surplus of B, and since the relation of the final import surplus of B to B's final income decline must (with no change in B's home investments) be equal to B's marginal propensity to save, the relation between the income increase of A and the income decline of B must be determined by the relation between the marginal propensity to save in B to the marginal propensity to save in A. With this proposition we have gone beyond observation and have started to deduce analytical relationships between the variables involved in the problem.

THE MULTIPLIER FORMULA INVOLVING PROPENSITIES IN TWO COUNTRIES

The simple formula generally given for the income multiplier does not include foreign repercussions. In the discussion

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[1] Let us try out these three ways of calculating ΔY in an example taken from Table IV-f. Find ΔY of country A for the fourth period. Method 1: $70.50 - 24.20 + 100.00 = 146.30$. Method 2: $141.00 + 33.50 - 28.20 = 146.30$. Method 3: $(100.00 + 50.00 + 37.00 + 33.50) - (0 + 20.00 + 26.00 + 28.20) = 220.50 - 74.20 = 146.30$.

[2] Thus, since in all six tables of Model IV s_A is assumed to be .2, the final change in the export surplus must be one-fifth of the final income change of A. Both the export surplus and the final income are, of course, affected by the foreign repercussions, which becomes quite obvious from the relationships stated in the next paragraph of the text above.

of Model II we expanded the formula to include a term f, standing for foreign repercussions, but we did not state how f was determined. This must be done now.

The multiplier formulas are summations of geometric progressions. These geometric progressions, rather simple as long as the income of the country concerned is independent of the income changes abroad, become very complicated when income changes in A affect incomes in B and, in turn, income changes in B have repercussions upon incomes in A. Exactly this is what happens.   If A's exports are B's imports, and A's imports are B's exports, incomes in each country are affected, period after period, by the foreign-induced changes in exports.   We are, thus, faced with interlocking geometric progressions.

These interlocking geometric progressions make it very hard to derive a formula for calculating the income level attained after any time interval, t, has elapsed, but the formula for the "full multiplier" is still free of the intricacies of the interlocking feature.   For, after an infinite number of periods, the incomes in each country must be of such magnitudes that the amounts saved or dissaved in each country tally with the foreign balance, which of course must be of equal size and opposite sign in the two countries.   Furthermore, since the differences between incomes in successive periods must have vanished—equilibrium having been reached—it will not matter that induced home consumption and induced imports are computed from current income rather than from those of the "preceding" period.

We may conveniently start from two propositions (the first of which has already been mentioned).   First: Since changes in home investment are ruled out and, hence, investment or disinvestment in foreign balances are the only possible offsets to saving and dissaving, and since the foreign balances, in the case of only two countries, must be of equal size and opposite

signs, the amounts saved and dissaved, respectively, in the two countries must be of equal size, and must also, in equilibrium, correspond to the respective propensities to save at the respective income levels. Thus (again omitting the Δ signs for increments) we may write that, after an infinite length of time,

$$(6) \qquad\qquad s_A Y_{A,\infty} = -s_B Y_{B,\infty}.$$

Second: Since the contributions to income in each period can be found to consist of two positive ones, namely the autonomous export (X) and the home-consumed portion of income, and a negative one, namely the foreign-induced reduction in export, and since the latter must correspond to the foreign propensity to import at the foreign income level, we can write

$$(7) \qquad\qquad Y_{A,\infty} = X + c_A Y_{A,\infty} + m_B Y_{B,\infty}.$$

(The last mentioned term is negative because Y_B is negative.)

From these two equations, we can obtain[1]

$$(8c) \quad Y_{A,\infty} = X \frac{1}{s_A + m_A + m_B \dfrac{s_A}{s_B}}; \qquad \text{or}$$

$$k_A = \frac{1}{s_A + m_A + m_B \dfrac{s_A}{s_B}}.$$

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[1] We write (6) in the form (6a) $Y_{B,\infty} = - \dfrac{s_A}{s_B} Y_{A,\infty}$. Substituting (6a) in (7) we obtain

$$(8) \qquad\qquad Y_{A,\infty} = X + c_A Y_{A,\infty} - m_B \frac{s_A}{s_B} Y_{A,\infty},$$

and, through transformation,

$$(8a) \qquad\qquad X = Y_{A,\infty} \left(1 - c_A + m_B \frac{s_A}{s_B}\right),$$

or

$$(8b) \qquad\qquad Y_{A,\infty} = X \frac{1}{1 - c_A + m_B \dfrac{s_A}{s_B}}$$

In the text we replace, in (8c), $1 - c_A$ by $s_A + m_A$.

In analogous fashion we obtain for country B

$$(8d) \quad Y_{B,\infty} = -X \frac{1}{s_B + m_B + m_A \dfrac{s_B}{s_A}}; \quad \text{or}$$

$$k_B = \frac{1}{s_B + m_B + m_A \dfrac{s_B}{s_A}}.$$

The last two formulas (8c and 8d) give us the final income changes as multiples of the autonomous change in foreign trade. Each multiplier contains as constants the four marginal propensities. The difference between these multipliers and the customary ones consists in the terms for the mutual repercussions. The foreign repercussion, which was formerly provisionally denoted as f, is found to be equal to the foreign marginal propensity to import multiplied by the ratio of the domestic to the foreign marginal propensity to save $\left(\text{e.g.,} f_A = m_B \dfrac{s_A}{s_B}\right)$.

EXPLANATION OF THE OBSERVED RELATIONSHIPS

We are now equipped to give explanations for most of the relationships observed in Tables IV-a–f.

From equation (6) above (expressing the equality of A's and B's net export or import balances, as the saved or dissaved portions of the respective income changes) we learn why the final income in A is to the final income in B as the marginal propensity to save in B is to the marginal propensity to save in A. ($Y_A : Y_B = k_A : k_B = s_B : s_A$). Thus, it is clear why the propensities to import, although influential upon the *magnitudes* of the multipliers, do not affect the *ratio* between the multipliers.

From our multiplier formulas we learn why the multiplier will be higher, the larger the marginal propensity to save is

in the foreign country, all other propensities being given. Fo r the larger the induced reductions in B's savings are, the more quickly will B's income decline be arrested and the less will be the decline of B's imports from A, that is, the less will be the check which reductions of A's exports exercise upon A's income increase.

Likewise, we learn why the multiplier will be lower the larger the marginal propensity to import is in the foreign country, all other propensities being given. For, the larger the induced declines in B's imports are, the more will A's income increase be checked through falling export business.

Summarizing the effects of all four propensities upon the foreign trade multiplier of a country, we can state that the multiplier will be higher (1) the smaller the marginal propensity to save is in the home country, (2) the smaller the marginal propensity to import is in the home country, (3) the larger the marginal propensity to save is in the foreign country, and (4) the smaller the marginal propensity to import is in the foreign country.

A quick glance at our multiplier formulas makes us understand why the multiplier in one country is not affected when the propensities to save and to import in the other country change in the same proportion. For, in the term describing the foreign repercussion upon A, there appears m_B in the numerator and s_B in the denominator.

What magnitudes of the foreign repercussion or what relationships between the propensities will depress the multiplier to unity or even less than unity? Another glance at numerator and denominator of our multiplier formulas will make it obvious that the multiplier in country A will be equal to one if the denominator, $s_A + m_A + \dfrac{s_A m_B}{s_B} = 1$, and that the multiplier will be less than one if $s_A + m_A + \dfrac{s_A m_B}{s_B} > 1$. When

will this be the case? We know that $s_A + m_A < 1$, because $s_A + m_A = 1 - c_A$. Hence, the multiplier will be equal to one if $\frac{s_A m_B}{s_B} = c_A$, that is, if the foreign repercussion is equal to the marginal propensity to consume home products. And the multiplier will be less than one if $\frac{s_A m_B}{s_B} > c_A$, that is, if the foreign repercussion is greater than the marginal propensity to consume home products.

Instead of saying $\frac{s_A m_B}{s_B} > c_A$ one may prefer to say that $\frac{m_B}{s_B} > \frac{c_A}{s_A}$ or that $\frac{s_A}{s_B} > \frac{c_A}{m_B}$. Putting the latter inequality in words, one may state that the foreign-trade multiplier in a country will be less than unity if the ratio between the domestic and foreign marginal propensities to save exceeds the ratio of the marginal propensity to consume home products to the foreign country's marginal propensity to import.

The explanation of the sequences leading to a new equilibrium level of income can best be commenced with our often repeated proposition that income goes on increasing as long as the autonomous increase in exports (Auton. ΔX) is in excess of induced increases in saving (ΔS) plus induced increases in imports (Home-ind. ΔM) plus induced decreases in exports (For.-ind. $-\Delta X$). Equilibrium is reached when $\Delta X_{auton.} = \Delta S_{ind.} + \Delta M_{ind.} - \Delta X_{ind.}$. During the transition period the differences between the autonomous side and the induced side of the equation will change the money and income flow in the country. It should be noted, however, that the mentioned difference (between the autonomous and the induced items) does not constitute the income increase as against the initial period but the income increase as against the preceding period. An excess of autonomous exports over the three induced items adds to the existing money and income flow

and, therefore, explains changes of the income increase from period to period. This will help us in the explanation of two things: (*a*) the mentioned "third method" of computing current income, and (*b*) the waves in the sequence of income changes.

(*a*) By "current income change" we have meant in most places the income change in comparison with the income of the initial period. (Hence, ΔY of the initial, i.e., zero, period was 0, and ΔY of the first period was $100.) Our "first method" of calculating this current income change used as basic data all disbursement items so that their algebraic sum would give the income change as against the initial level. ($\Delta X_{auton.,t} + \Delta C_{ind.,t} + \Delta X_{ind.,t} = \Delta Y_t$.) The "third method" used saving and foreign balances as basic data. Current saving and the current foreign balance will not give us disbursement figures; but the difference between the two gives us the amounts of monetary expansion or contraction as against the immediately preceding period:

$$(\Delta X_{auton.,t} - \Delta M_{ind.,t} + \Delta X_{ind.,t}) - \Delta S_{ind.,t} = \Delta Y_t - \Delta Y_{t-1}.$$

Cumulating the expansionary and deflationary doses over the whole time so far elapsed, we obtain the current income change as against the initial level.[1]

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[1] Another relationship can be made clear by the same line of reasoning. It can be observed that at any time, during transition as well as after equilibration, the difference between the income increase in *A* and the income decrease in *B* will be equal to the difference between the accumulated amounts of induced saving in *A* and cumulated amounts of induced dissaving in *B*. (For this observation I am indebted to Mr. Joseph Ullman.) The explanation is as follows: In country *A* the increase in income is made up of all the successive increments in the income flow which are due, in each period, to the excess of the export surplus over the increase in saving; in country *B* the decrease in income is made up of all the successive decrements in the income flow which are due, in each period, to the excess of the import surplus over the reduction of saving. Since *A*'s export surplus and *B*'s import surplus are an identical magnitude, the sum of all savings in *A* and the sum of all dissavings in *B* over the elapsed periods must

(b) Now to the waves in the sequence of income changes. As long as the three induced items together fall short of the autonomous item, the income increase is continually growing.   When the induced items together equal the autonomous item, income has reached its stable equilibrium level.   But if the induced items surpass the autonomous item, there must be a reversal in the sequence of incomes.   Under what conditions will it happen that the autonomous export will be surpassed by the sum of induced saving, induced import and the induced fall in export?   The answer suggests itself if one reflects on the facts that income changes are the sources, and the various propensities are the forces, of the "inducements" concerning the three items in question.   If all propensities involved added up to less than one, and if they were all operating on the basis of home income alone, the induced amounts together could never rise above the autonomous export. However, one of the induced items has not home income but foreign income as its source; this foreign-induced item will operate with greater power if the multiplier in the foreign country is higher than the multiplier at home; in this case the foreign income decline will surpass the domestic income increase and, therefore, the foreign-induced fall in exports may sooner or later become heavy enough to overcompensate, along with induced saving and induced import, the expansionary effect of the autonomous export.   Hence, income, after having grown to some peak, will recede to a more moderate level.

Thus we understand that the country with the lower multiplier, that is, the country with the higher marginal propensity to save, will be subject to a fluctuation in the movement of its income toward the new equilibrium level.   If it is the importing country which has the higher marginal propensity to save

determine at any time the difference between the changes in the income levels of the two countries.

and, therefore, the lower multiplier, it will be there that the wave in the income sequence will be observed. The income there will decline to some low point but will then recover to a less depressed level.

Although a wave in the income sequence will cause, as has been mentioned above, a wave in the sequence of induced imports, this will not cause a wave to occur also in the sequence of net foreign balances. For it is the heavy foreign-induced decrease (or increase) in exports which causes the wave in the income sequence, which, in turn, causes a wave in the sequence of increases (decreases) of, home-induced imports; the foreign-induced decrease (increase) in the exports will therefore be large enough to drown the wave in the home-induced import sequence. Hence, the net export surplus (import surplus) will continually fall from the peak in the first period—i.e., the amount of the autonomous export (import)—to the equilibrium level.

THE CONTINUAL EXPORT SURPLUS

The equilibrium level of the foreign balance is, in all cases of Model IV, definitely above the zero level. In other words, the export surplus of country A and the import surplus of country B, although greatly reduced from the "favorable" and "unfavorable" balance, respectively, of the first period, will not completely disappear. From the $100 of the autonomous change in foreign trade, the trade balance settled down to a "final" level which ranged, in the six cases exhibited above, from $14.28 (Table IV-d) to $36.73 (Table IV-e).

Why does the "automatic rectification" of the trade balance fail to take place in these cases, (at least if, according to the accepted assumptions, all prices, interest rates and foreign-exchange rates remain unchanged)? The trade balance cannot "rectify" itself because the *induced* changes in foreign trade cannot rise to the level of the *autonomous* change in

foreign trade if induced saving and dissaving, uncompensated by changes in home investment, interfere. With induced saving in country A and dissaving in B, contributing to the arresting of the increase and decrease, respectively, of incomes in A and B, equilibrium will be reached while favorable and unfavorable trade balances continue to prevail. Foreign lending and borrowing (or gold movements) will be the equilibrating counter-items of the export and import surpluses.

As has been explained above (see the section on "Induced Foreign Lending"), the continuance of an export surplus will not operate as an expansionary force if it is offset by increased thrift. If, for instance, the banking system of the exporting country acquires foreign balances, or grants loans to customers who acquire foreign assets, this expansion may be offset by a reduction in other assets which "induced savers" acquire from the banking system. Or, the bank expansion may be offset by an accumulation of idle balances on the part of savers.

Thus, the trade balance will not rectify, and still less, reverse itself. Country B will continue to lose gold or foreign assets, or increase its foreign indebtedness, as long as no autonomous import reductions or export increases occur. B politicians who dislike a loss of gold or an increase in foreign indebtedness will, of course, clamor for protective measures such as higher import duties, export subsidies, exchange restrictions or exchange depreciation. An anti-mercantilist would be loath to accept the argument but (if prices, interest rates, home investment, etc. are assumed to remain unchanged) he must admit that the monetary contraction that follows the import increase in B will fail to rectify the trade balance. The home-induced decline in imports and the foreign-induced increase in exports together are not strong enough to wipe out the unfavorable trade balance; the contraction of income invites a reduction in saving and this reduced

thrift "offsets" the loss of gold and foreign assets or the increase in debts to foreigners.   Hence, the continuance of the unfavorable trade balance does not involve a continuance of the monetary contraction.   Although domestic bank deposits are lost through a loss of gold (reserves) or balances in foreign banks, or through an increase of deposits of foreigners, it may turn out that the lost domestic deposits are fully replaced through increased domestic bank loans, the demand for which derives from the reduced thrift.   Or it may be dishoarding which offsets a further decline in deposits.

The size of the ultimately remaining export or import surplus is not a function of the *magnitudes* of the marginal propensities to save or import but a function of the *proportions* between these propensities.   On first thought one might suppose that the equilibrium level of the foreign balance, since it must be equal to the induced saving and dissaving in the two countries, must depend on the propensities to save.   This would be mistaken.   After all, the equilibrium level of saving is not merely a simple function of the propensity to save but depends also on the level of income on which that propensity has a chance to operate.   And this level of income is affected by the marginal propensity to save as well as by the marginal propensity to import and the foreign repercussion.

The ultimate export surplus of country A (identical with the import surplus of country B) is, we repeat, with unchanged home investment, equal to the equilibrium rate of saving. Thus,

$$(9) \qquad X_{A,\infty} - M_{A,\infty} = s_A Y_{A,\infty}.$$

With the $Y_{A,\infty}$ known from our multiplier formula $(8c)$, we obtain[1] for A's ultimate export surplus

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[1] According to our multiplier formula,

$$(10a) \qquad X_{A,\infty} - M_{A,\infty} = X \frac{1}{1 + \dfrac{m_A}{s_A} + \dfrac{m_B}{s_B}}.$$

We see now clearly that the ultimate export surplus will vary with $\dfrac{s_A}{m_A}$ and with $\dfrac{s_B}{m_B}$. The export surplus of country A, and the import surplus of country B, will be greater, the greater A's marginal propensity to save is in relation to A's marginal propensity to import and also the greater B's marginal propensity to save is in relation to B's marginal propensity to import.[1]

THE TIME FACTOR IN THE REVISED MULTIPLIER

Before we developed the multiplier formula involving foreign repercussions we examined the simple multiplier as a function of time. (See above p. 49 ff.) We arrived at

$$(8c) \qquad \Upsilon_{A,\infty} = X \frac{1}{s_A + m_A + m_B \dfrac{s_A}{s_B}}.$$

Substituting (8c) in (9), we obtain

$$(10) \qquad X_{A,\infty} - M_{A,\infty} = X \frac{s_A}{s_A + m_A + m_B \dfrac{s_A}{s_B}}.$$

Dividing numerator and denominator by s_A, we get (10a) as written in the text above.

[1] From equation (10a) we can draw the following inferences. If the marginal propensity to consume home products rises at the expense of the import propensity (either in A or B or in both countries) the final export surplus will be greater. If the propensity to consume home products rises, however, at the expense of the propensity to save, the final export surplus will be smaller. If the propensity to consume home products as well as the propensity to import both rise at the expense of the propensity to save, the final export surplus will likewise be smaller. But if the propensity to consume home products rises at the expense of both saving and import, then the export surplus may rise or fall or remain unchanged, according as the reduction in the saving propensity is lesser or greater than, or the same as, the reduction in the import propensity. (For this note I am indebted to Mr. Ullman.)

formulas giving (*a*) the income level attained after certain time intervals have elapsed, and (*b*) the time interval which it takes for income to increase to a level within 10 per cent of the "full multiple." The formulas were relatively simple functions of *c*, the marginal propensity to consume. (See equations (3) and (5).) But we announced great difficulties for the corresponding solutions in the case of foreign repercussions.

If foreign repercussions are taken into consideration, the task of finding equations which give us the income increases attained, not after an infinite time interval, but after *t* periods have elapsed, leads us into a maze of interlocking progressions. And the outcome, unfortunately, is not altogether satisfactory, because the equations which can be produced are clumsy and unwieldy.

In order to avoid frightening the reader who shies from looking at long series of equations, the development of the multiplier formula as a function of time is corralled in the appendix. (Appendix A.) But so much can be said at this place that the formula is derived from equations which express the income of any period as determined by certain disbursements out of the incomes of the preceding period. By combining three such recursion formulas an equation is obtained which expresses the income of a period as a function of the constant propensities and of the time elapsed since the autonomous change occurred.

With this formula (see p. 223), which is not reproduced here because of its forbidding looks, we have found the solution to one of the problems raised in this section. But we have set ourselves still another task, namely, to find a general formula giving us the *t*, the number of time periods, after which nine-tenths of the final income will have been realized. Unfortunately, this task appears to be insoluble. One can, by purely arithmetic means and the use of tabulated values of

exponentials, work out tables of the t's of the ninety-per cent-full multiples for various sets of values of the propensities.[1] But a general formula giving the t as a determinate function of the propensities cannot be obtained algebraically.

In the cases where wavelike movements of the changes in income occur, the respective figures may once pass through the ten-per cent neighborhood of equilibrium[2] and then leave this zone again in order to return there only on the way back. For example, in Table IV-a, we find the income increase of country A in the second period within 10 per cent of the ultimate multiple, but then from the third to the ninth period outside this zone. Only in the tenth period, that is, after something like 27–30 months, do the figures of the generated income reenter the neighborhood area.

In the six examples which we have gone through, the adjustment period—i.e., the period in which a point within 10 per cent from the equilibrium level is reached—took between 2 and 11 income propagation periods, that is, between 3 and 33 months.[3]

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[1] This was suggested to me by Professor Paul A. Samuelson. I did not produce such tables, because the demands on effort and space would be disproportionate to their usefulness. Since each table could take at best two propensities as variables and would have to take the other two propensities as parameters, too many such tables would be needed for a satisfactory array of combinations.

[2] I borrow the term "neighborhood of equilibrium" from Professor Joseph A. Schumpeter and hope I shall be forgiven if I use it not with all the connotations which he assigned to it in his work on *Business Cycles*, New York, 1939, p. 71.

[3] The generated income does not increase in jumps (like the fare in a taxicab) but more or less continuously. For this reason, I translated "2 periods" into calendar time as if it read "after the end of the first period."

Chapter VI

THE FIFTH, SIXTH AND SEVENTH MODELS: SEVERAL COUNTRIES. THE REACTIONS IN A THIRD COUNTRY. THE MULTI-PLIER FORMULA INVOLVING THREE COUNTRIES

THE WORLD HAS MORE THAN TWO COUNTRIES

The world as we know it has more than two countries, a fact which complicates, apart from many other things, the theory of the multiplier. But we must "face reality" and, therefore, admit the existence of other countries besides A and B. This concession to realism may be considered by some as an unnecessary step in our theory, for, it may be held, all the other foreign countries besides B might be considered as parts of B. If the effects of increased green cheese exports upon A's national income are examined, what difference does it make whether the importers are in country B with many other countries lying around or whether the importers are in a province B_1 which is a part of a very large country B? In other words, may not all foreign countries be rolled into one and regarded as *the* foreign country?

While such a device is quite permissible in the analysis of many problems, it does not seem to be helpful in the theory of the foreign-trade multiplier. Various reasons militate against the use of the device of merging all foreign nations in our theory. In any case, it should be clear that the "import" propensity of a union of countries taken as a whole must be

much smaller than the sum of "import" propensities of all its parts taken separately. For, obviously, trade between regions is international trade if the regions are separate countries but intranational trade if they are parts of one country. The propensity to purchase goods from another region would be "propensity to import" in the one case, but "propensity to consume domestic products" in the other. (If the whole world were one country its import propensity would be zero.)

Assuming we knew the marginal propensities to import of all countries, we would not know the marginal propensity to import in a union of all countries but one. For now only the propensity to buy imports from that one separate country would be relevant. Hence, for each combination of countries the import propensity would be different.

To assume a *given* marginal propensity to import for the super-country B which comprises all countries except A, would be rather misleading; for, in order to be relevant for the sequence of changes, the marginal import propensity of the imaginary union would depend on which "province" would feel first the impact of increased imports and reduced incomes; some "provinces" may have relatively large, others relatively small, imports from the one independent nation (A) which, because it is the very object of his analysis, the theorist allows to exist. Rather than part with the assumption that the import propensities are independent of the direction and composition of the autonomous trade changes, we had better put up with the existence of more countries.[1]

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[1] To assume equal income-propagation periods appeared to me on first thought less tenable in the united-country case than in the two-country case; the assumption that a round of spending takes equally long in two countries of about equal size is, of course, highly artificial; it seemed still worse to make this assumption if one of the countries were nearly the whole world. But, after all, it would make no difference whether the assumption was made for each of several separate countries or for two, one of which comprised the whole lot. So there was no escape from this dilemma. If I understand it correctly, Professor Jacob Viner's concept of the "final pur-

Thus, we introduce more foreign countries, with the proviso, however, that they exhibit such characteristics that our theorizing will not be unduly complicated.   Let us assume first that there are ten countries besides A, so that country B, the importer of A's green cheese, is one among these ten foreign countries.   All countries are of approximately equal importance in the trade relations among one another. Hence, A's marginal propensity to import is equally distributed over the products of all ten countries; and likewise, if a change in B's income affects its purchases from abroad, A's industries will feel just one-tenth of that total change. Since the nine other countries are all alike and distinguish themselves from A and B only by the fact that the autonomous change in trade is confined to the cheese business between A and B, we shall not have to borrow too large a part of the alphabet: all the nine countries are just like country C.

MODEL V: SEVERAL FOREIGN COUNTRIES, EQUAL PROPENSITIES IN A AND B

Country C (and the others) will cause absolutely no complications if we assume, for this model only, that the propensities to save and to import in country B are equal to the corresponding propensities in country A.   In this case, country C will simply "drop out" without giving rise to any repercussions.   This is so for the following reason.   If both the saving and import propensities are the same in A and B, the income increase in A, resulting from the autonomous export increase to B, will be exactly of the same magnitude as the income decrease in B.   These income changes affect imports.   The home-induced increase in imports of A will be exactly equal to the home-induced decrease in imports of B.   That is to say, A will buy more imports from C, and B will buy less

chases velocity of circulation" is pertinent here.   (*Studies in the Theory of International Trade*, p. 369 ff.)

imports from C. Country C will therefore experience a change in the direction of its trade but no change in the volume or in the balance of trade. The export trade of C (and of the other eight countries) will merely alter its destination and perhaps its composition, all lost exports to "impoverished" B being replaced by new exports to "enriched" A. Since incomes in C and the other eight countries are, therefore, not affected, their purchases of imports from A or B will not be affected either. Hence, neither A nor B will feel any repercussions following their increased or decreased purchases from C and the others.

The facts, however, that the home-induced increase of imports of A and the home-induced decrease of imports of B are both spread over ten countries rather than concentrated on one, and that these changes cancel out in nine of the ten countries, indicate that the "foreign repercussion" is reduced to one-tenth of the magnitude it had in the two-country case.[1] When incomes decline in B in consequence of its cheese purchases from A, and B's effective demand for imports is thereby impaired, only one-tenth of its import reduction will hit back at A. And no "triangular repercussions" will follow. The foreign-induced fall in exports of A is thus only one-tenth of what it would be if no third countries were involved.

Table V gives the sequence of events in countries A and B. The propensities to save are assumed to be .2, the propensities to import .3, in both A and B. What the propensities are in country C and the rest is immaterial because incomes there are not affected. What distinguishes the sequences shown in Table V from those in Tables IV-a–f is that the home-

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[1] The larger the number of foreign countries the smaller will be the foreign repercussion upon country A. Another way of stating this, in line with the country-merger device, would be the following. The larger the world outside of A, regarded as one super-country made up of all foreign countries, the smaller would be the marginal propensity to import in this super-country and, therefore, the smaller the foreign repercussion upon country A.

induced changes of imports of one country are no longer identical with the foreign-induced changes of exports of the other country, but that the latter are now only one-tenth of the former. (The figures in columns 5 and 13 are $\frac{1}{10}$ of those in columns 12 and 4, respectively.)

A comparison of the results of this case with those of the case shown in Table IV-b suggests itself here, because all assumed propensities are the same in the two cases. In case IV-b, where there were no third countries, the ultimate

	COUNTRY A : s=0.2 M=0.3								COUNTRY B : s=0.2 M=0.3							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	K=1.887				Foreign Balance			Income from 2+5+6	K=1.887				Foreign Balance			Income from 10+13-14
	Spendable Income				=¹⁄₁₀ of 12	=14	=-15		Spendable Income				=¹⁄₁₀ of 4	=6	=-7	
Period	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Auton. ΔX	Net Δ(X-M)	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Auton. ΔM	Net Δ(X-M)	Current ΔY
1						100.00	100.00	100.00						+100.00	-100.00	-100.00
2	100.00	50.00	20.00	30.00	-3.00	100.00	67.00	147.00	-100.00	-50.00	-20.00	-30.00	+3.00	+100.00	-67.00	-147.00
3	147.00	73.50	29.40	44.10	-4.41	100.00	51.49	169.09	-147.00	-73.50	-29.40	-44.10	+4.41	+100.00	-51.49	-169.09
4	169.09	84.54	33.82	50.73	-5.07	100.00	44.20	179.47	-169.09	-84.54	-33.82	-50.73	+5.07	+100.00	-44.20	-179.47
5	179.47	89.73	35.90	53.84	-5.38	100.00	40.78	184.35	-179.47	-89.73	-35.90	-53.84	+5.38	+100.00	-40.78	-184.35
6	184.35	92.17	36.87	55.31	-5.53	100.00	39.16	186.64	-184.35	-92.17	-36.87	-55.31	+5.53	+100.00	-39.16	-186.64
7	186.64	93.32	37.33	55.99	-5.59	100.00	38.42	187.73	-186.64	-93.32	-37.33	-55.99	+5.59	+100.00	-38.42	-187.73
8	187.73	93.86	37.55	56.32	-5.63	100.00	38.05	188.23	-187.73	-93.86	-37.55	-56.32	+5.63	+100.00	-38.05	-188.23
∞	188.67	94.33	37.74	56.60	-5.66	100.00	37.74	188.67	-188.67	-94.33	-37.74	-56.60	+5.66	+100.00	-37.74	-188.67

TABLE V.—Autonomous export from A to B. Nine other countries involved. Equal propensities in A and B.

multiplier was 1.25; in case V, where there are ten foreign countries, the multiplier is 1.887. In other words, the increase of income in the export country, and the decrease of income in the import country, are greater if through the presence of third countries a good part of the foreign repercussion is eliminated.

THE MULTIPLIER WITH DIMINISHED FOREIGN REPERCUSSION

The reduction of the foreign repercussion through the "intervention" of third countries must be made explicit in

another revision of the multiplier formula. In the last version of the multiplier formula for country A, the foreign repercussion was expressed by the term $m_B \dfrac{s_A}{s_B}$ added to the denominator. Now, with several countries involved, we know that of the mentioned term only a fraction equal to the ratio which the one importing country (B) bears to the total number of foreign countries can operate as foreign repercussion. That is to say, if there are n foreign countries besides B, only $\dfrac{1}{n+1}$ of the foreign repercussion of the two-country case will be felt.

Let us bear in mind that we have been dealing here with the case where the propensities in A and B were equal, for it is only in this case that the propensities in country C do not matter at all. We shall see at the next model, that is, in the case where the propensities in A and B are not equal but proportional $\left(\text{i.e., } \dfrac{s_A}{s_B} = \dfrac{m_A}{m_B}\right)$, that there the propensities in C will matter only during the transition period but will not matter in the end. In both these cases, therefore, the ultimate foreign repercussion upon country A will be only $\dfrac{m_B}{n+1}\dfrac{s_A}{s_B}$. Hence the ultimate income in country A will be

$$(20) \qquad Y_{A,\infty} = X \frac{1}{s_A + m_A + \dfrac{m_B}{n+1}\dfrac{s_A}{s_B}};$$

and in country B

$$(20a) \qquad Y_{B,\infty} = X \frac{1}{s_B + m_B + \dfrac{m_A}{n+1}\dfrac{s_B}{s_A}}.$$

If, as in Model V, $s_A = s_B$ and $m_A = m_B$, the multipliers will reduce to

(21)

$$k_{A,\infty} = k_{B,\infty} = \frac{1}{s_A + m_A \left(1 + \dfrac{1}{n+1}\right)} = \frac{1}{s_A + m_A \dfrac{n+2}{n+1}}.$$

(For example, under the assumptions of Table V, $k_{A,\infty} = k_{B,\infty} = \dfrac{1}{\frac{2}{10} + \frac{3}{10} \cdot 1\frac{1}{10}} = \dfrac{100}{53} = 1.8867.$)

Instead of assuming a certain number of countries all of which have the same trade relations among one another, it may be preferable to correct the foreign repercussion by a coefficient which shows more directly the percentage which the B-import from A constitutes of all countries' imports from A. Then we need not assume that there are "nine countries like C, which all have the same volume of trade with A that B has." Instead of saying, as we would under this assumption, that $n = 9$ and only $\dfrac{1}{n+1} \left(= \dfrac{1}{10} \right)$ of $m_B \dfrac{s_A}{s_B}$ will appear in A's multiplier, we can simply say that B has been purchasing 10 per cent of A's exports and therefore $\frac{1}{10}$ of $m_B \dfrac{s_A}{s_B}$ will operate as a leakage in A's generated income flow.

Let us call $x_{A,B}$ the ratio between A's exports to B and the total of A's exports; and $x_{B,A}$ the ratio between B's exports to A and the total of B's exports. Then we can write the multipliers as follows:

(22)

$$k_{A,\infty} = \frac{1}{s_A + m_A + x_{A,B} \cdot m_B \dfrac{s_A}{s_B}};$$

and

(22a)

$$k_{B,\infty} = \frac{1}{s_B + m_B + x_{B,A} \cdot m_A \dfrac{s_B}{s_A}}.$$

If $s_A = s_B$ and $m_A = m_B$, we can write

$$(23) \qquad k_{A,\infty} = \frac{1}{s_A + m_A(1 + x_{A,B})}$$

It should be noted that if country B's import from A approaches a negligible fraction of A's total exports, the multiplier will approach the value it would have under the simple formula neglecting foreign repercussions altogether.[1]

MODEL VI: SEVERAL FOREIGN COUNTRIES, PROPORTIONAL PROPENSITIES IN A AND B

When the propensities to save and to import are not the same in the exporting as in the importing country, the changes in incomes and the induced changes in imports of the two countries will not be of equal magnitudes and the effects on third countries will not cancel out. Hence, incomes in country C (and the rest) will change and its imports will change accordingly. And, at least throughout the transition period, the change in C's imports will act back upon the exports of A and B.

It will be interesting to observe not merely the events in countries A and B but also what happens in country C. For this reason we shall maintain the assumption that C is one of nine similar countries and that A, B, C and the other eight are all, apart from the cheese business between A and B, of equal importance in the trade relations among one another. Thus, any home-induced change in imports of any one

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[1] The results of this analysis were anticipated in the brilliant essay by William A. Salant (cited above in footnote 1, p. 25). Cf. this statement of his: "The repercussions on a country's exports of a change in its imports will be greater, other things being equal, if the countries which supply most of its imports are also the principal customers for its exports." (*Op. cit.*, p. 214.)

country will affect the exports of the other ten countries with equal force, that is, each of the other ten countries will experience a foreign-induced change in exports amounting to one tenth of the said import change.

For reasons which will become clear later we begin with assuming that the propensities to save and to import in B are proportional to the respective propensities in A. Thus let us assume that the marginal propensity to save is .2 in country A and .1 in B; and that the marginal propensity to import is .3 in A and .15 in B. The propensities in C and the rest are assumed to be equal to those in B, though this is of no special significance.

Table VI is constructed on this basis. Somewhat complicated items in this table are the foreign-induced changes of exports. As in all cases so far discussed, country A loses exports because of the home-induced reduction of imports in B; but A gains some exports because of the home-induced increase of imports in C and the rest. A's gain of exports to C alone amounts to $\frac{1}{10}$ of the import increase in C; but since there are nine countries such as C, A has new exports in an amount equal to $\frac{9}{10}$ of the import increase in C; the loss of exports to B amounts to $\frac{1}{10}$ of B's home-induced reduction of imports.

The same reasoning explains the foreign-induced changes in B's exports. These will amount to $\frac{1}{10}$ of the home-induced imports in A plus 9 times $\frac{1}{10}$, i.e. $\frac{9}{10}$, of the home-induced imports in C. Here both changes are positive because, in the case under examination, incomes increase in C as well as in A.

Country C gains exports in an amount of $\frac{1}{10}$ of A's home-induced increase in imports, and loses exports in an amount of $\frac{1}{10}$ of B's home-induced decrease in imports; but it gains also exports to the other eight countries in a total amount of 8

times $\frac{1}{10}$ of the size of its own home-induced increase in imports (because the other eight countries import the same amount as C does).[1]

The correctness of the calculated foreign balances of each period can be easily checked, because at any interval of time the sum of all export surpluses must be equal to the sum of all import surpluses.[2] In Table VI the sum of the positive trade balance of country A plus 9 times the positive trade balance of country C must be equal, in each period, to the negative trade balance of country B. (To obtain perfect checks of the balances the figures were first calculated for more than two decimal places.)

Let us now go through the sequence of events in country C. No autonomous change in trade occurs in that country. The first thing that happens is, in the second period, an increase in exports which is foreign-induced, that is induced

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[1] For example, the foreign-induced changes in exports in the third period are computed in the following way:

Country A		Country B	
$\frac{1}{10}$ of home-ind. ΔM_B	-2.5800	$\frac{1}{10}$ of home-ind. ΔM_A	$+4.4550$
$\frac{9}{10}$ of home-ind. ΔM_C	$+.2025$	$\frac{9}{10}$ of home-ind. ΔM_C	$+.2025$
For.-ind. ΔX_A	-2.3775	For.-ind. ΔX_B	$+4.6575$

Country C	
$\frac{1}{10}$ of home-ind. ΔM_A	$+4.4550$
$\frac{1}{10}$ of home-ind. ΔM_B	-2.5800
$\frac{8}{10}$ of home-ind. ΔM_C	$+.1800$
For.-ind. ΔX_C	$+2.0550$

[2] It may be worth noting that this "equality" of favorable and unfavorable trade balances is really an *identity* and should be distinguished from the equality of net savings and favorable trade balances, which (in absence of home investment and with our definition of saving) is only an *equilibrium condition,* hence an equality which will be realized only when incomes attain their equilibrium levels.

COUNTRY A: S = 0.2 M = 0.3 — K = 1.8867 — Foreign Balance — Income from 2+5+6 (columns 1–8)
Col 4 = %o of 12, %o of 20; Col 6 = 14; Col 7 = −15 −(9×23)

COUNTRY B: S = 0.1 M = 0.15 — K = 3.7736 — Foreign Balance — Income from 10+13−14 (columns 9–16)
Col 12 = %o of 4, %o of 20; Col 14 = 6; Col 15 = −7 −(9×23)

COUNTRY C: S = 0.1 M = 0.15 — Foreign Balance — Income from 18+21 (columns 17–24)
Col 20 = %o of 4, %o of 20; Col 23 = −15−7 ⁄ 9

Period	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Auton. ΔX	Net Δ(X·M)	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Auton. ΔM	Net Δ(X·M)	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Auton. ΔX	Net Δ(X·M)	Current ΔY
1						100.00	100.00	100.00						+100.00	−100.00	−100.00						–		
2	100.00	50.00	20.00	30.00	−1.50	100.00	68.50	148.50	−100.00	−75.00	−10.00	−15.00	+3.00	+100.00	−82.00	−172.00					1.50	–	1.50	1.50
3	148.50	74.25	29.70	44.55	−2.38	100.00	53.07	171.87	−172.00	−129.00	−17.20	−25.80	+4.66	+100.00	−69.54	−224.34	1.50	1.12^{5}	.15	$.22^{5}$	2.05^{5}	–	1.83	3.18
4	171.87	85.94	34.37	51.56	−2.94	100.00	45.50	183.00	−224.34	−168.26	−22.43	−33.65	+5.59	+100.00	−60.67	−262.67	3.18	2.38	.32	.48	2.17	–	1.69	4.56
5	183.00	91.50	36.60	54.90	−3.32	100.00	41.78	188.18	−262.67	−197.00	−26.27	−39.40	+6.11	+100.00	−54.49	−290.89	4.56	3.42	.46	.68	2.10	–	1.41	5.52
6	188.18	94.09	37.64	56.45	−3.62	100.00	39.93	190.47	−290.89	−218.17	−29.09	−43.63	+6.39	+100.00	−49.98	−311.78	5.52	4.14	.55	.83	1.94	–	1.11	6.08
7	190.47	95.24	38.09	57.14	−3.86	100.00	39.00	191.38	−311.78	−233.83	−31.18	−46.77	+6.53	+100.00	−46.70	−327.30	6.08	4.56	.61	.91	1.77	–	.85	6.33
8	191.38	95.69	38.28	57.41	−4.06	100.00	38.53	191.63	−327.30	−245.48	−32.73	−49.09	+6.60	+100.00	−44.31	−338.88	6.33	4.75	.63	.95	1.59	–	.64	6.34
9	191.63	95.82	38.32	57.49	−4.23	100.00	38.28	191.59	−338.88	−254.16	−33.89	−50.83	+6.61	+100.00	−42.56	−347.55	6.34	4.75	.64	.95	1.43	–	.48	6.18
10	191.59	95.80	38.31	57.48	−4.38	100.00	38.14	191.42	−347.55	−260.66	−34.76	−52.13	+6.58	+100.00	−41.29	−354.08	6.18	4.63	.62	.93	1.28	–	.35	5.91
11	191.42	95.71	38.28	57.43	−4.51	100.00	38.06	191.20	−354.08	−265.56	−35.41	−53.11	+6.54	+100.00	−40.35	−359.02	5.91	4.43	.59	.89	1.14	–	.25	5.57
12	191.20	95.60	38.24	57.36	−4.63	100.00	38.01	190.97	−359.02	−269.27	−35.90	−53.85	+6.49	+100.00	−39.66	−362.78	5.57	4.18	.56	.83	1.02	–	.19	5.20
13	190.97	95.49	38.19	57.29	−4.74	100.00	37.97	190.75	−362.78	−272.08	−36.28	−54.42	+6.43	+100.00	−39.15	−365.65	5.20	3.90	.52	.78	.91	–	.13	4.81
14	190.75	95.37	38.15	57.23	−4.83	100.00	37.94	190.54	−365.65	−274.24	−36.56	−54.85	+6.37	+100.00	−38.78	−367.87	4.81	3.61	.48	.72	.81	–	.09	4.42
15	190.54	95.27	38.11	57.16	−4.92	100.00	37.92	190.35	−367.87	−275.90	−36.79	−55.18	+6.31	+100.00	−38.51	−369.59	4.42	3.32	.44	.66	.73	–	.07	4.05
16	190.35	95.18	38.07	57.10	−5.00	100.00	37.90	190.18	−369.59	−277.19	−36.96	−55.44	+6.26	+100.00	−38.30	−370.93	4.05	3.03	.41	.61	.65	–	.04	3.68
17	190.18	95.09	38.04	57.05	−5.07	100.00	37.88	190.02	−370.93	−278.20	−37.09	−55.64	+6.20	+100.00	−38.16	−372.00	3.68	2.76	.37	.55	.58	–	.03	3.34
18	190.02	95.01	38.00	57.01	−5.13	100.00	37.86	189.88	−372.00	−279.00	−37.20	−55.80	+6.15	+100.00	−38.05	−372.85	3.34	2.51	.33	.50	.52	–	.02	3.03
∞	188.67	94.33	37.74	56.60	−5.66	100.00	37.74	188.67	−377.36	−283.02	−37.74	−56.60	+5.66	+100.00	−37.74	−377.36	0	0	0	0	0	0	0	0

TABLE VI.—Autonomous export from A to B.   Nine other countries involved.   Proportional propensities in A and B.

by foreign income changes. The increase in A's import demand, originating from increased income in A, exceeds the decrease in B's import demand, which results from decreased income in B, and thus C acquires the favorable trade balance. This balance, amounting to $1.50, generates additional income in the same amount. And, in the next (the third) period, this available income is put to the following uses: $.15 are saved, $1.125 are spent on domestic consumption goods, and $.225 are spent on imports. At the same time a further increase in exports occurs, most of the new exports going to the prospering country A, but some of them going to the other eight, unnamed countries; the new exports substantially overbalance the lost exports to B. The net foreign-induced exports and the additional home consumption of this period make for new income in an amount of $3.18.

In the fourth period, foreign-induced exports are still rising, but home-induced imports are rising by more, so that the export surplus will be smaller than in the preceding period. In the fifth period, foreign-induced exports begin to fall (largely because imports in A have reduced their rate of increase while imports in B still continue to fall heavily) and, with C's home-induced imports rising further, the export surplus falls lower. Incomes, however, continue their upward movement for three more periods, reaching their peak at $6.34 in the eighth period.

Home-induced imports must now begin to fall, but since they fall more slowly than foreign-induced exports, the export surplus diminishes seriously. Incomes fall slowly though consistently. In the eighth period the peak income of $6.34 was associated with an export surplus of $.64; in the eighteenth period an income of $3.03 goes with an export surplus of less than· $.02. The trade balance does not completely turn around and become negative; it merely vanishes. And

since the leakage through saving then finds no offset whatsoever, the generated income will also gradually disappear. Thus all the items in C which were generated through the trade between A and B are approaching zero.

MULTIPLIER, INCOMES AND SAVINGS IN THE THIRD COUNTRY

Before we inquire into the question why in this case country C, after actively participating in the play of interactions and repercussions, finally drops out of the picture altogether without participating in the equilibrium constellation, we should take notice of the moral which our observations contain for investigators in the statistical derivation of the multiplier. Country C obtained a favorable trade balance and an increase in income.   A statistician, set upon calculating the multiplier of country C from trade and income figures, would get the queerest results.   If, in Keynesian disregard for time lags, simultaneous data were compared, the "multiplier" would grow fiercely: it would be almost 4 in the 5th period (the fourth period from the first occurrence of the export surplus), almost 10 in the 8th period, 22 in the 11th period, 49 in the 14th period, 151 in the 18th period; and it would rise to still loftier heights before the figures from which the multiplier is calculated would vanish.

Even if C's cumulated export surplus over its whole life time, that is over the whole adjustment period, were set in relation to the aggregate income generated over the same period, the "multiplier" would be fantastically high.   And, of course, this multiplier would have very little to do with its usual relation, the marginal propensity to consume.

All this is not really surprising, for we know that country C has had no autonomous increase in exports; we know that

its increase in exports was merely a reflection of trade-induced income changes in other countries. But how should the statistician know?

It would be possible to derive a formula which gives as a function of time the income changes in C which result from the autonomous export from A to B. The development of such a formula, however, would have little use, save as a mathematical exercise. The ultimate income change in C, in this case where A and B have proportional propensities to save and to import, was zero. Now, why did incomes in C, generated through the effects of the trade between A and B, disappear again when equilibrium establishes itself?

The answer can best and most conclusively be furnished by way of algebraic reasoning, but it should also be possible to rely on merely literary common sense.

Country C (which stands here for all countries except A and B) can have an export surplus (import surplus) only so long as the export surplus of A falls short of (exceeds) the import surplus of B. If incomes in the countries reach equilibrium in a situation in which B's import surplus is equal to A's export surplus, then, obviously, C can have no positive or negative foreign balance.

Equilibrium in A is reached when the autonomous exports are completely offset by induced savings, induced imports and induced reductions in exports.[1] Equilibrium in B is reached when the autonomous imports are completely offset by induced reductions in saving, induced reductions in imports and induced increases in exports. These equilibrium conditions can also be expressed by saying that A's savings must be equal to its export surplus, and B's negative savings must be equal to its import surplus. Hence, in an equilibrium

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[1] In our symbols this condition reads: auton. ΔX_A = ind. ΔS_A + home-ind. ΔM_A − for.-ind. ΔX_A (with home investment always assumed unchanged).

in which country C is no longer involved, the four items—saving in A, negative saving in B, the export surplus of A and the import surplus of B—must all be equal.

Now, from previous reasoning (in the two-country case) we know that the final income increase in A will vary, not with the magnitudes of, but with the proportions between, the marginal propensities to save and import in B; and the final income decrease in B, likewise, will vary with the proportions between the propensities in A. Furthermore (see pp. 86–87), the final level of saving in each country will vary with the ratio between the propensities to save and to import in the home country and with the corresponding ratio in the foreign country. This ratio in "the foreign country," determining the foreign repercussion, will, in the case of more than two countries, be combined with variables pertaining to country C. Now, what is the home propensities ratio for A is the foreign ratio for B; and what is the home ratio for B is the foreign ratio for A. If, and only if, the two ratios are equal, the variables pertaining to C will have the same end-effect on A and B, and the savings in A must equal the dis-savings in B, and, likewise, the export surplus of A must equal the import surplus of B, just as in the two-country case. Thus, in the case where the propensities in B are exactly proportional to those in A, country C, will end up with unchanged trade balance and income.[1]

In Table VI income was generated in country C, rose to a peak, and then receded gradually, eventually to disappear. It goes without saying that the opposite movement of incomes

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[1] I must confess that I have had great difficulties in formulating this merely verbal reasoning and, if I have succeeded at all (of which I am not too sure), I attribute it to previous algebraic reasoning. Here is one of the cases where a statement of literary economics was obtained only through translation from mathematical groundwork.

in C is equally possible. If the marginal propensity to import is greater in the importing country, B, than in the exporting country, A, the decline in B's imports will initially exceed the increase in A's imports, and country C will lose exports. The net loss of exports in C will cause a decline in income; income in C will fall to a trough and then gradually recover. In this case with proportional propensities in A and B, C's import surplus will vanish soon and income will eventually go back to the predisturbance level.

During the transition period country C, in the case of an export surplus, must have accumulated some savings, or, in the case of an import surplus, must have sustained negative savings. What form would these savings or dissavings have taken? These questions were dealt with previously in another context, so that they can be quickly answered here. Since home investments were assumed to remain unchanged, the savings (dissavings) of the transition period must leave country C, when equilibrium is reestablished, with more (less) gold and foreign claims or less (more) foreign debts. Of course, it would be foolish to assume that the savers themselves, for mysterious reasons, would have acquired the foreign assets or would have repaid foreign debts; or, in the opposite case, that the dissavers themselves would have disposed of foreign assets or would have contracted foreign debts. Those whose foreign assets or debts have changed in the process are most probably the Monetary Authority, or banks in general, while savers and dissavers have acquired or lost domestic assets, or repaid or contracted domestic debts. The easiest way of visualizing the situation is, in the case of the temporary export surplus (e.g. the case shown in Table VI), to assume that the banks have acquired foreign assets such as deposits in foreign banks—or that the banks have acquired reserves through gold purchases by the Monetary

Authority—while they have lost (to the savers) an equal amount of domestic assets.

MODEL VII: SEVERAL FOREIGN COUNTRIES, NON-PROPORTIONAL PROPENSITIES IN A AND B

With proportional marginal propensities to save and to import in countries A and B, the changes of trade balances and incomes in C and other countries were merely temporary. If the propensities in A and B are not proportional, the changes of trade balances and incomes in C and other countries do not disappear in equilibrium. The autonomous change in trade between A and B will have lasting effects upon trade and incomes in C and the other countries. And, of course, the repercussions from C, and the rest, upon countries A and B will make the determination of the end-effects in A and B through multiplier formulas much more complicated than in the simpler cases treated so far.

That Model VII is in terms of *three* foreign countries, as against the ten foreign countries which were participating in Models V and VI, is of no significance other than that of providing variety. In the former two models, n was 9; now $n = 2$.

We are going to work out two cases; one where the import propensity of country B falls short of that of A, so that country C will acquire an export surplus (Table VII-a); another where the import propensity of B exceeds that of A, so that C will sustain an adverse trade balance (Table VII-b). Both tables are carried through many more income periods than would seem necessary merely for the observation of the income increase in country A, because other series of figures in A or the other countries continue to exhibit interesting changes

long after the income in A has entered the zone of the ten-per cent-neighborhood of equilibrium.[1]

With altogether four countries present, each country has to deal with three other countries. The change of the imports of any one country is equally distributed over the three foreign countries. Thus, each country sustains changes in exports which result from the income changes in the three others. The foreign-induced changes of exports in country A are: a loss equal to one third of B's import reduction, and a gain (Table VII-a) or loss (Table VII-b) equal to two thirds of C's import change. (Two thirds—because there is another country like C.) The foreign-induced changes of exports in B are: a gain equal to one third of A's import increase, and a gain (VII-a) or loss (VII-b) equal to two thirds of C's import change. The foreign-induced changes of exports in C are: a gain equal to one third of A's import increase, a loss equal to one third of B's import reduction, and a gain (VII-a) or loss (VII-b) equal to one third of its own import change (because the third, or fourth, country has the same import change that C has itself).

The correctness of the computed figures can again be tested by checking whether the algebraic sum of all trade balances of each period is zero. In Table VII-a the export surplus of A plus twice the export surplus of C must be equal to B's import surplus. In Table VII-b the export surplus of A must be equal to the sum of B's import surplus plus twice that of C.

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[1] For example, in Table VII-a, the income increase in A attains the level of the ninety-per cent-full multiple in the fifth period; but the income series in B has a turning point in the sixth period, causing several other series to start movements in reverse. In Table VII-b, the income in A reaches the equilibrium neighborhood zone in the fourth period; but the export series of A and several other series in B and C pass through turning points in the seventh period.

In Table VII-a the propensities are the following: $s_A = .2$, $m_A = .3$; $s_B = .3$, $m_B = .2$; $s_C = .2$, $m_C = .3$. Incomes in A and in C can be observed to increase continuously, while income in B exhibits a wave, falling first to a low point and then recovering slightly. The export surplus in C does not stay at the level reached in the fourth and fifth periods, but recedes, though almost imperceptibly. Income in C goes on rising, because savings, though consistently increasing, stay below the export surplus, approaching it only by slow degrees.[1] The export surplus of A, starting with the $100.00 of the autonomous exports, shrinks to below $40 after four periods and finally settles down at a level above $37. The import surplus of B falls from the initial $100 less rapidly and settles down at a level near $46. Although the ultimately remaining import surplus of B is larger than the remaining export surplus of A, the final income in B shows a decrease smaller than the increase of the final income in A.

We may compare the multipliers in case VII-a with those in case IV-f, for in both cases the marginal propensities to save and to import in A and B are identical. In the two-country case $k_{A,\infty}$ was 1.579 and $k_{B,\infty}$ was 1.053. In case VII-a, with four countries involved, the multiplier[2] in A is 1.8803, and the multiplier in B is 1.5385. Of course, the more countries there are, the smaller will be the leakages and seepages through foreign repercussions and, therefore, the larger will be the multipliers.

The propensities assumed for case VII-b are $s_A = .2$, $m_A = .3$; $s_B = .2$, $m_B = .4$; $s_C = .3$, $m_C = .3$. The extremely flat troughs of the income waves in B and C are peculiar. In B we find income in the third period reduced by $139.00,

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[1] That the final level of foreign-induced exports in country C is the same as the level of current domestic consumption is due merely to the fact that the latter in this case is $\frac{5}{10}$ of the income. In equilibrium $\Delta S + \Delta M$ must be equal to ΔX because ΔS must be equal to $\Delta X - \Delta M$.

[2] The ultimate values were calculated with the help of the formulas developed in the next section.

Period	COUNTRY A: S=0.2 M=0.3								COUNTRY B: S=0.3 M=0.2								COUNTRY C: S=0.2 M=0.3							
	K=1.8803				Foreign Balance			Income from	K=1.5385				Foreign Balance			Income from					Foreign Balance			Income from
	Spendable Income				$\frac13$ of 12 $\frac23$ of 20 =14	=14	-15 -(2×23)	2+5+6	Spendable Income				$\frac13$ of 4 $\frac23$ of 20 =6	=6	-7 -(2×23)	10+13+14	Spendable Income				$\frac13$ of 4 $\frac13$ of 12 $\frac13$ of 20		-7-15 /2	18+21
	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Auton. ΔX	Net Δ(X-M)	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Auton. ΔM	Net Δ(X-M)	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Auton. ΔX	Net Δ(X-M)	Current ΔY
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1						100.00	100.00	100.00						+100.00	-100.00	-100.00								
2	100.00	50.00	20.00	30.00	-6.67	100.00	63.33	143.33	-100.00	-50.00	-30.00	-20.00	+10.00	+100.00	-70.00	+40.00					3.33	-	3.33	3.33
3	143.33	71.66	28.67	43.00	-8.67	100.00	48.33	163.00	-140.00	-70.00	-42.00	-28.00	+15.00	+100.00	-57.00	+155.00	3.33	1.66	.67	1.00	5.33	-	4.33	7.00
4	163.00	81.50	32.60	48.90	-8.93	100.00	42.17	172.57	-155.00	-77.50	-46.50	-31.00	+17.70	+100.00	-51.30	+159.80	7.00	3.50	1.40	2.10	6.67	-	4.57	10.17
5	172.57	86.28	34.51	51.77	-8.62	100.00	39.61	177.66	-159.80	-79.90	-47.94	-31.96	+19.29	+100.00	-48.75	+160.61	10.17	5.08	2.03	3.06	7.62	-	4.57	12.71
6	177.66	88.83	35.53	53.30	-8.16	100.00	38.54	180.66	-160.61	-80.31	-48.18	-32.12	+20.31	+100.00	-47.57	+160.00	12.71	6.36	2.54	3.81	8.33	-	4.52	14.68
7	180.66	90.33	36.13	54.20	-7.73	100.00	38.07	182.60	-160.00	-80.00	-48.00	-32.00	+21.00	+100.00	-47.00	+159.00	14.68	7.34	2.94	4.40	8.86	-	4.46	16.20
8	182.60	91.30	36.52	54.78	-7.36	100.00	37.86	183.94	-159.00	-79.50	-47.70	-31.80	+21.50	+100.00	-46.70	+158.00	16.20	8.10	3.24	4.86	9.28	-	4.42	17.38
9	183.94	91.97	36.79	55.18	-7.06	100.00	37.76	184.91	-158.00	-79.00	-47.40	-31.60	+21.87	+100.00	-46.53	+157.13	17.38	8.69	3.48	5.21	9.60	-	4.38	18.29
10	184.91	92.46	36.98	55.47	-6.81	100.00	37.71	185.64	-157.13	-78.57	-47.14	-31.42	+22.15	+100.00	-46.43	+156.41	18.28	9.14	3.66	5.49	9.85	-	4.36	18.99
11	185.64	92.82	37.13	55.69	-6.63	100.00	37.68	186.13	-156.41	-78.21	-46.92	-31.28	+22.36	+100.00	-46.39	+155.84	18.99	9.49	3.80	5.70	10.03	-	4.33	19.52
∞	188.03	94.01	37.61	56.41	-5.98	100.00	37.61	188.04	-153.85	-76.92	-46.16	-30.77	+23.07	+100.00	-46.16	+153.85	21.37	10.69	4.27	6.41	10.68	-	4.27	21.37

TABLE VII-a.—Autonomous effort from A to B. Two other countries involved. Non-proportional propensities in A and B.

reaching the low point, or greatest decline, of $142.44 in the sixth period, but stagnating near this level from then on, to be stabilized eventually at $142.01. In C the income decline hovers around $6 from the fourth period on. It is also interesting to see the foreign balances practically stabilized in spite of continuing slight changes in exports and imports; these changes are largely balancing each other, thus leaving net balances unchanged. Incidentally, the ultimate import surplus of country C is very small in this case: a mere $1.78; and the import surplus of B has settled down at $28.39.

Comparing cases VII-a and VII-b we find that in the latter the final multipliers are lower in both countries. Instead of the 1.8803 of the former case, the multiplier in A is now only 1.5976. Responsible for this lower multiplier is obviously the higher import propensity in B, which makes the foreign-induced loss of exports in A more serious.

The rule which we learned in the two-country case, that the final changes of income in A and B will bear to each other a proportion exactly reciprocal to the marginal propensities to save—a rule which still held in the case of several countries if the propensities to save and to import were proportional in B to those in A—this rule is no longer valid when the propensities in A and B are not proportional and the repercussions of other countries become permanent. In case VII-b the propensities to save are equal in A and B, but the magnitudes of the changes in income are not equal. The relationships are now more complicated, as will be seen from the form of the new multiplier formula which covers cases conforming to Model VII.

THE MULTIPLIER FORMULA INVOLVING PROPENSITIES IN THREE COUNTRIES

In the last revision of the multiplier formula we introduced one or several "third" countries, but there it was only the number of these countries, not their propensities which

TABLE VII-b. Several Countries Involved

	COUNTRY A: s=0.2 M=0.3 K=1.5976								COUNTRY B: s=0.2 M=0.4 K=1.4201								COUNTRY C: s=0.3 M=0.3							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
		Spendable Income			Foreign Balance			Income from 2+5+6		Spendable Income			Foreign Balance			Income from 10+13-14		Spendable Income			Foreign Balance			Income from 18+21
Period	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX (⅓ of 12, ⅓ of 20)	Auton. ΔX (=14)	Net Δ(X·M) (-15 -(2×23))	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX (⅓ of 4, ⅓ of 20)	Auton. ΔM (=6)	Net Δ(X·M) (-7 -(2×23))	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Auton. ΔX	Net Δ(X·M) (-7-15 /2)	Current ΔY
1						100.00	100.00	100.00						+100.00	-100.00	-100.00								
2	100.00	50.00	20.00	30.00	-13.33	100.00	56.67	136.67	-100.00	-40.00	-20.00	-40.00	+10.00	+100.00	-50.00	-30.00					-3.33	—	-3.33	-3.33
3	136.67	68.34	27.33	41.00	-18.00	100.00	41.00	150.34	-130.00	-52.00	-26.00	-52.00	+13.00	+100.00	-35.00	-139.00	-3.33	-1.33	-1.00	-1.00	-4.00	—	-3.00	-5.33
4	150.34	75.17	30.07	45.10	-19.60	100.00	35.30	155.57	-139.00	-55.60	-27.80	-55.60	+13.97	+100.00	-30.43	-141.63	-5.33	-2.13	-1.60	-1.60	-4.03	—	-2.43	-6.17
5	155.57	77.79	31.11	46.67	-20.12	100.00	33.21	157.67	-141.63	-56.65	-28.33	-56.65	+14.32	+100.00	-29.03	-142.33	-6.17	-2.47	-1.85	-1.85	-3.94	—	-2.09	-6.41
6	157.67	78.84	31.53	47.30	-20.26	100.00	32.44	158.58	-142.33	-56.93	-28.47	-56.93	+14.49	+100.00	-28.58	-142.44	-6.41	-2.56	-1.93	-1.93	-3.85	—	-1.92	-6.41
7	158.58	79.29	31.72	47.57	-20.27	100.00	32.16	159.02	-142.44	-56.98	-28.49	-56.98	+14.58	+100.00	-28.44	-142.40	-6.41	-2.56	-1.93	-1.93	-3.77	—	-1.84	-6.33
8	159.02	79.51	31.80	47.71	-20.26	100.00	32.03	159.25	-142.40	-56.96	-28.48	-56.96	+14.63	+100.00	-28.41	-142.33	-6.33	-2.53	-1.90	-1.90	-3.72	—	-1.82	-6.25
9	159.25	79.63	31.85	47.77	-20.23	100.00	32.00	159.40	-142.33	-56.93	-28.47	-56.93	+14.67	+100.00	-28.40	-142.66	-6.25	-2.50	-1.88	-1.88	-3.68	—	-1.80	-6.18
10	159.40	79.70	31.88	47.82	-20.20	100.00	31.98	159.50	-142.26	-56.90	-28.45	-56.90	+14.71	+100.00	-28.39	-142.19	-6.18	-2.47	-1.85	-1.85	-3.65	—	-1.80	-6.12
11	159.50	79.75	31.90	47.85	-20.18	100.00	31.97	159.57	-142.19	-56.88	-28.44	-56.88	+14.73	+100.00	-28.39	-142.15	-6.12	-2.45	-1.84	-1.84	-3.62	—	-1.78	-6.07
12	159.57	79.79	31.91	47.87	-20.16	100.00	31.97	159.63	-142.15	-56.86	-28.43	-56.86	+14.75	+100.00	-28.39	-142.11	-6.07	-2.43	-1.82	-1.82	-3.60	—	-1.78	-6.03
13	159.63	79.81	31.93	47.89	-20.15	100.00	31.96	159.66	-142.11	-56.84	-28.42	-56.84	+14.76	+100.00	-28.39	-142.08	-6.03	-2.41	-1.81	-1.81	-3.59	—	-1.78	-6.00
∞	159.76	79.88	31.95	47.93	-20.12	100.00	31.95	159.76	-142.01	-56.81	-28.39	-56.81	+14.80	+100.00	-28.39	-142.01	-5.92	-2.36	-1.78	-1.78	-3.55	—	-1.78	-5.92

TABLE VII-b.—Autonomous export from A to B. Two other countries involved. Non-proportional propensities in A and B.

mattered, for equal or proportional propensities in A and B were assumed. The formula for the foreign-trade multiplier must now be adapted to the case where the marginal propensities to save and import in B are not proportional to those in A and where, therefore, the propensities to save and import in C will, through lasting repercussions upon A and B, affect the end-results in all countries.

This newly revised formula of the foreign-trade multiplier can be obtained from three simultaneous equations expressing the equilibrium conditions in the three countries, that is to say, the eventual equality, in any one country, of the changes in saving to the changes in the foreign balance (with home investment assumed unchanged). Again the assumption is made that all countries concerned are of equal importance in the trade relations with one another, and that all countries but A and B are in every respect identical with C.

The equilibrium condition in A is expressed by

$$(24) \quad s_A Y_{A,\infty} = X - m_A Y_{A,\infty} + \frac{1}{n+1} m_B Y_{B,\infty} + \frac{n}{n+1} m_C Y_{C,\infty}.$$

The four terms on the right side of the equation constitute all the factors shaping the final export surplus of A, namely, the autonomous export, the induced increase in imports from foreign countries, the induced decrease in exports to B (decrease, because $Y_{B,\infty}$ is negative) and the induced increase (or possibly decrease) in exports to C and the rest.

Likewise we write as equilibrium conditions for B and C, respectively,

$$(25) \quad s_B Y_{B,\infty} = -X - m_B Y_{B,\infty} + \frac{1}{n+1} m_A Y_{A,\infty} + \frac{n}{n+1} m_C Y_{C,\infty},$$

and

$$(26) \quad s_C Y_{C,\infty} = \frac{1}{n+1} m_A Y_{A,\infty} + \frac{1}{n+1} m_B Y_{B,\infty} + \frac{n-1}{n+1} m_C Y_{C,\infty} - m_C Y_{C,\infty}.$$

Writing $k_A X$ for $\Upsilon_{A,\infty}$, and $-k_B X$ for $\Upsilon_{B,\infty}$, we can solve the three equations for k_A and k_B. This is done in Appendix B.[1]

The resulting formulas are clumsy. In order to abbreviate them somewhat, we introduce a new term, γ, for a repeatedly recurring expression, containing the propensities of C, namely,

$$(29) \qquad \gamma = \frac{n}{n+1} \frac{m_C}{s_C(n+1) + 2m_C}$$

With this term, we can obtain by steps reproduced on Appendix B our new version of the foreign-trade multipliers for the exporting and the importing country,

$$(32d) \quad k_A = \frac{s_B + m_B\left(\dfrac{n}{n+1} - 2\gamma\right)}{[s_A + m_A(1-\gamma)][s_B + m_B(1-\gamma)] - m_A m_B\left(\dfrac{1}{n+1} + \gamma\right)^2},$$

and

$$(33) \quad k_B = \frac{s_A + m_A\left(\dfrac{n}{n+1} - 2\gamma\right)}{[s_A + m_A(1-\gamma)][s_B + m_B(1-\gamma)] - m_A m_B\left(\dfrac{1}{n+1} + \gamma\right)^2}.$$

The variables contained in these expressions are s_A, m_A, s_B, m_B, and, through the γ, s_C, m_C, and n. The formulas constitute, of course, the full multipliers, that is to say, the ones which, operating on the autonomous change in foreign trade, give the "ultimate" income, which is generated (or destroyed after an infinite length of time). The formulas certainly have undergone a considerable development from their modest beginnings, when they had the innocent looks of a mere

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[1] Equations are numbered consecutively in the appendix; numbers between (26) and (32d) which are omitted here are only in the appendix.

$\frac{1}{1 - c_A}$. Yet, formidable as they look, they are easily handled when numbers are substituted for all variables.[1]

Instead of the full multipliers, we may prefer to have formulas giving incomes as a function of time; we have supplied such formulas for the two-country cases, and there is little doubt that for "practical" purposes short-period results would be more valid than full-adjustment results. I did derive formulas for this foreign-trade multiplier as a function of time, but they are so long and terrifying that I dare not reproduce them.[2]

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[1] The ultimate values in Tables VII-a and VII-b were calculated from the formulas. For example, for $s_A = .2$, $m_A = .3$, $s_B = .3$, $m_B = .2$, $s_C = .2$, $m_C = .3$, and $n = 2$, as assumed for Table VII-a,

$$\gamma = \frac{2}{3} \cdot \frac{.3}{.2(3) + 2(.3)} = \frac{2}{3} \cdot \frac{.3}{1.2} = \frac{1}{6},$$

$$k_A = \frac{.3 + .2(\tfrac{2}{3} - \tfrac{1}{3})}{[.3 + .2(\tfrac{5}{6})][.2 + .3(\tfrac{5}{6})] - (.3)(.2)(\tfrac{1}{3} + \tfrac{1}{6})^2} = \frac{.3666}{.195} = 1.88034,$$

and

$$k_B = \frac{.2 + .3(\tfrac{2}{3} - \tfrac{1}{3})}{.195} = \frac{.3}{.195} = 1.53846.$$

The ultimate values for country C were then calculated from equation (26c), given in the appendix.

[2] I derived them from the following seven equations:

$$(34.1) \quad Y_{A,t} = X + c_A Y_{A,t-1} + \frac{1}{n+1} m_B Y_{B,t-1} + \frac{1}{n+1} m_C Y_{C,t-1}$$

$$(34.2) \quad Y_{A,t-1} = X + c_A Y_{A,t-2} + \frac{1}{n+1} m_B Y_{B,t-2} + \frac{1}{n+1} m_C Y_{C,t-2}$$

$$(34.3) \quad Y_{A,t-2} = X + c_A Y_{A,t-3} + \frac{1}{n+1} m_B Y_{B,t-3} + \frac{1}{n+1} m_C Y_{C,t-3}$$

$$(34.4) \quad Y_{B,t-1} = -X + c_B Y_{B,t-2} + \frac{1}{n+1} m_A Y_{A,t-2} + \frac{1}{n+1} m_C Y_{C,t-2}$$

$$(34.5) \quad Y_{B,t-2} = -X + c_B Y_{B,t-3} + \frac{1}{n+1} m_A Y_{A,t-3} + \frac{1}{n+1} m_C Y_{C,t-3}$$

$$(34.6) \quad Y_{C,t-1} = c_C Y_{C,t-2} + \frac{1}{n+1} m_A Y_{A,t-2} + \frac{1}{n+1} m_B Y_{B,t-2} + \frac{n-1}{n+1} m_C Y_{C,t-2}$$

$$(34.7) \quad Y_{C,t-2} = c_C Y_{C,t-3} + \frac{1}{n+1} m_A Y_{A,t-3} + \frac{1}{n+1} m_B Y_{B,t-3} + \frac{n-1}{n+1} m_C Y_{C,t-3}$$

Chapter VII

THE EIGHTH MODEL: NOT AUTONOMOUS BUT FOREIGN-INDUCED EXPORTS. THE FOREIGN-INDUCED-TRADE MULTIPLIER. THE HOME-INVESTMENT MULTIPLIER WITH FOREIGN REPERCUSSIONS

INDUCED CHANGES IN EXPORTS

Up to this point the analysis has been exclusively on the effects of autonomous changes in foreign trade. Induced changes in imports and exports were an integral part of the analysis but only in so far as they were attributable to previous autonomous changes in foreign trade. Thus, if we started with an increase in exports from country A to country B, we made certain that it was caused by a change in tariffs, transport costs, production technique or consumers' tastes. An increase in exports from A to B resulting from increased incomes in country B would not be an autonomous change, according to our chosen terminology.

To a superficial observer this might appear as terminological stubbornness. If country A's export business in green cheese is increased, what difference should it make whether the people in B buy more of this imported delicacy because they have suddenly developed a taste for it, or because they find that the import duties were lowered, or because they have higher incomes? It does make a difference. If people in B buy more A-cheese because, at reduced import duties, they prefer it to their home-made cheese and give up buying the latter,

incomes in B are going to fall and the B-people's purchases of other imports from A must fall accordingly. On the other hand, if people in B buy more A-cheese because for some reason they are getting higher incomes, the increased imports may somewhat check the national income in B from rising further but will not reduce it absolutely; and hence, other purchases from A need not fall. Thus, we must keep the foreign-income-induced exports apart from those additional exports which do not originate from income changes.

The analogous reasoning holds for reduced exports. It is one thing if A's exports are reduced because of import restrictions in country B. It is another if A's exports are reduced because of shrinking incomes in country B. The effects of these reductions in exports upon the national income in A are not the same; that is to say, the multipliers are different. For these reasons we must insist on separating the two types of change in export, calling the one an autonomous change, the other a foreign-induced change in export.

MODEL VIII: AUTONOMOUS INVESTMENTS IN B, INDUCED EXPORTS OF A

In analyzing the effects of a foreign-induced change in exports we must start with the income change abroad which affected the foreign demand for imports. Thus, if there is an increase in exports from A to B, and the effects upon the national income of A are to be studied, we have to go back to the expansion of expenditures in B. These additional expenditures, financed (in the beginning at least) through dishoarding or credit creation, may be for consumption or for home investment or both.

We avoid here the questions whether autonomous increases in private consumption are likely occurrences and whether autonomous increases in public consumption should or should not be called investment ("honorary investment"); we simply

assume that things begin with additional expansionary invest-
ment, public or private, in country B. It is further assumed
that no imported materials are needed for the investment
itself. But, with the marginal propensity to import greater
than zero, the income generated through the investment in B
must lead to increased imports of consumers' goods or services,
or materials for consumers' goods, from country A. A's
exports, thus increased, generate income in A and, in turn,
lead to induced imports of A. And, of course, with positive
marginal propensities to save in both countries, induced sav-
ings occur in A as well as in B.

This sequence is shown in our Model VIII, which is limited
again to only two countries. Induced home investment is
again ruled out and our whole familiar list of assumptions
is maintained. We begin with the same set of marginal
propensities with which we set out in Model IV. Thus Table
VIII-a is based upon $s_A = .2$, $m_A = .3$, $s_B = .1$ and $m_B = .15$.
Country B has its income expanded through autonomous
investments of $100.00 per period. With a marginal propen-
sity to import 15 per cent of each received increment of income,
B will purchase from A $15.00 worth of goods in the second
period and increasing amounts thereafter, increasing until
the rise of incomes in B is estopped. Incomes in A, generated
by these foreign-induced exports, are partly used for buying
imports from B; hence, the new export surpluses of A increase
less than its exports. A's induced imports—a foreign reper-
cussion from the point of view of B—contribute to the genera-
tion of income in B. Thus, the leakage in B, checking the
rise of its income, is somewhat counteracted by the foreign-
induced exports. While induced saving and induced imports
are retarding, the foreign-induced exports are energizing
factors in the development of incomes in B.

On account of the large marginal propensity to consume
home-made goods in country B [$c_B = .75$], it takes a very long

time[1] for incomes and trade volumes to approach an equilibrium level. No less than 17 periods, or more than four years, are needed for the adjustment to the ten-per cent-neighborhood of equilibrium of the income increments in *B*, and a still longer time for those in country *A*. Exports in country *A* are

Columns 1–8 = **COUNTRY A: S = .2, m = .3**; Columns 9–16 = **COUNTRY B: S = 0.1, m = 0.15**. For Country B, K = 6.25. Under "Foreign Balance": For-Ind ΔX of A "= 12", Net Δ(X-M) of A "= -14"; For-Ind ΔX of B "= 4", Net Δ(X-M) of B "= -6". Current ΔY of A = income from columns 2+5; Current ΔY of B = income from columns 10+13+15.

Period	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX (=12)	Net Δ(X-M) (=-14)	Auton ΔI	Current ΔY (2+5)	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX (=4)	Net Δ(X-M) (=-6)	Auton ΔI	Current ΔY (10+13+15)
1															100.00	100.00
2					15.00	15.00		15.00	100.00	75.00	10.00	15.00		-15.00	100.00	175.00
3	15.00	7.50	3.00	4.50	26.25	21.75		33.75	175.00	131.25	17.50	26.25	4.50	-21.75	100.00	235.75
4	33.75	16.88	6.75	10.12	35.36	25.24		52.24	235.75	176.81	23.58	35.36	10.12	-25.24	100.00	286.93
5	52.24	26.12	10.45	15.67	43.04	27.37		69.16	286.93	215.20	28.69	43.04	15.67	-27.37	100.00	330.87
6	69.16	34.58	13.83	20.75	49.63	28.88		84.21	330.87	248.15	33.09	49.63	20.75	-28.88	100.00	368.90
7	84.21	42.11	16.84	25.26	55.33	30.07		97.44	368.90	276.68	36.89	55.33	25.26	-30.07	100.00	401.94
8	97.44	48.72	19.49	29.23	60.29	31.06		109.01	401.94	301.46	40.19	60.29	29.23	-31.06	100.00	430.69
9	109.01	54.51	21.80	32.70	64.60	31.90		119.11	430.69	323.02	43.07	64.60	32.70	-31.90	100.00	455.72
10	119.11	59.55	23.82	35.73	68.36	32.63		127.91	455.72	341.79	45.57	68.36	35.73	-32.63	100.00	477.52
11	127.91	63.96	25.58	38.38	71.63	33.25		135.58	477.52	358.14	47.75	71.63	38.38	-33.25	100.00	496.52
12	135.58	67.79	27.12	40.68	74.48	33.80		142.27	496.52	372.39	49.65	74.48	40.68	-33.80	100.00	513.06
13	142.27	71.13	28.45	42.68	76.96	34.28		148.09	513.06	384.80	51.31	76.96	42.68	-34.28	100.00	527.48
14	148.09	74.05	29.62	44.43	79.12	34.69		153.17	527.48	395.61	52.75	79.12	44.43	-34.69	100.00	540.04
15	153.17	76.58	30.63	45.95	81.01	35.06		157.59	540.04	405.03	54.00	81.01	45.95	-35.06	100.00	550.98
16	157.59	78.79	31.52	47.28	82.65	35.37		161.44	550.98	413.23	55.10	82.65	47.28	-35.37	100.00	560.51
17	161.44	80.72	32.29	48.43	84.08	35.65		164.80	560.51	420.38	56.05	84.08	48.43	-35.65	100.00	568.81
∞	187.50	93.75	37.50	56.25	93.75	37.50		187.50	625.00	468.75	62.50	93.75	56.25	-37.50	100.00	625.00

TABLE VIII-a.—Investments increased in B. Foreign-induced exports of A. Induced saving and imports.

of course moving in step with the income increments in *B*. But the export surplus of *A* comes more quickly closer to its new equilibrium level: with $33.78 in the 12th period it is within 10 per cent of the final $37.50.

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[1] The larger the marginal propensity to consume domestic goods, the longer does it take for the adjustment to come into the 10-per cent-neighborhood of equilibrium. See above pp. 51 and 68, and Appendix A. *Cf.* also my article in the *Quarterly Journal of Economics*, Vol. 54, 1939, p. 15.

A FAMILY OF MULTIPLIERS

The incomes in Table VIII-a rise eventually by $625.00 in country B and $187.50 in country A, or by $812.50 in both countries together. In relation to the autonomous investment of $100.00, the composite income multiplier is between the 10 that would correspond to the marginal propensity to save of .1 in country B and the 5 that would correspond to the saving propensity of .2 in country A. However, one does not usually calculate "composite multipliers" for two or more countries together. For each country separately a variety of multipliers can be calculated, depending on what should be considered as the proper multiplicand (and, again, depending on the time intervals that one wishes to allow).

From the point of view of clean methodology, which would demand that induced changes be not shoved back into the multiplicand, the only acceptable multiplier for country A would be that relating the income increments in A to the autonomous investment of B $[\Delta Y_A \div \Delta I_B]$. Yet, this would be too strange a ratio to work with. More customary would it be to compare the income increment in A with the increment in either its exports or its export surplus. Looking at these ratios we find, for the equilibrium values, multipliers of 2 and 5 respectively. The one is the reciprocal of the sum of the marginal propensities to save and to import, the other is the reciprocal of the marginal propensity to save. $[\Delta Y_A \div \Delta X_A = 2$, because of $\dfrac{1}{.2 + .3}$; $\Delta Y_A \div \Delta(X_A - M_A) = 5$, because of $\dfrac{1}{.2}.]$ Thus we see here that the multipliers in the case of foreign-induced exports are unaffected by those "foreign repercussions" which have entered into the multiplier in the case of autonomous exports.[1]

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[1] The reason for this is fairly obvious: the foreign-induced change in exports, a part of the multiplier when autonomous changes in trade are

TABULATION OF VARIOUS MULTIPLIERS FOR THE VALUES OF TABLE VIII-a

	Full Equilibrium Values	Current "Quarterly" Statistics			"Annual" and "Bi-annual" Statistics	
		1st Period*	5th Period	17th Period	First Four Periods*	First Eight Periods*
$\Delta Y_A \div \Delta I_B$	$\dfrac{187.50}{100.00} = 1.88$	$\dfrac{15.00}{100.00} = .15$	$\dfrac{67.83}{100.00} = .68$	$\dfrac{164.64}{100.00} = 1.65$	$\dfrac{162.07}{400.00} = .41$	$\dfrac{569.12}{800.00} = .71$
$\Delta Y_A \div \Delta X_A$	$\dfrac{187.50}{93.75} = 2.00$	$\dfrac{15.00}{15.00} = 1.00$	$\dfrac{67.83}{42.84} = 1.58$	$\dfrac{164.64}{84.01} = 1.96$	$\dfrac{162.07}{119.45} = 1.36$	$\dfrac{569.12}{348.38} = 1.63$
$\Delta Y_A \div \Delta (X_A - M_A)$	$\dfrac{187.50}{37.50} = 5.00$	$\dfrac{15.00}{15.00} = 1.00$	$\dfrac{67.83}{27.84} = 2.44$	$\dfrac{164.64}{35.63} = 4.62$	$\dfrac{162.07}{91.18} = 1.78$	$\dfrac{569.12}{213.22} = 2.67$
$\Delta Y_B \div \Delta I_B$	$\dfrac{625.00}{100.00} = 6.25$	$\dfrac{100.00}{100.00} = 1.00$	$\dfrac{329.18}{100.00} = 3.29$	$\dfrac{568.42}{100.00} = 5.68$	$\dfrac{796.33}{400.00} = 1.99$	$\dfrac{2322.48}{800.00} = 2.90$
$\Delta Y_B \div \Delta (I_B + X_B)$	$\dfrac{625.00}{156.25} = 4.00$	$\dfrac{100.00}{100.00} = 1.00$	$\dfrac{329.18}{115.00} = 2.86$	$\dfrac{568.42}{148.38} = 3.83$	$\dfrac{796.33}{413.27} = 1.93$	$\dfrac{2322.48}{902.63} = 2.57$
$\Delta Y_B \div \Delta (I_B + X_B - M_B)$	$\dfrac{625.00}{62.50} = 10.00$	$\dfrac{100.00}{100.00} = 1.00$	$\dfrac{329.18}{72.16} = 4.56$	$\dfrac{568.42}{64.37} = 8.83$	$\dfrac{796.33}{336.66} = 2.37$	$\dfrac{2322.48}{618.65} = 3.75$

* The "first" period in country A is the second period from the occurrence of the autonomous investment in B. Likewise, the "first four" and "first eight" periods in country A are one period behind those in country B.

For country B we are not concerned with foreign-trade multipliers but, instead, with home-investment multipliers. The ratio of the eventual income increment in B to the autonomous investment $[\Delta Y_B \div \Delta I_B]$ is 6.25. However, if one prefers to compute—as statistical investigators would be most inclined to do—the combined effects of home investments and exports upon national income and, thus, if one takes both these items into the multiplicand, the resulting multiplier $[\Delta Y_B \div \Delta(I_B + X_B)]$ is 4. Finally, if home investment minus import surplus is taken as multiplicand, the multiplier $[\Delta Y_B \div \Delta(I_B + X_B - M_B)]$ is 10.

All these multipliers refer to the equilibrium values attained after a lag of infinite duration and approached within 10 per cent after more than four years. The levels attained in a shorter time differ, of course, considerably from the "final" ones. In view of the necessity of visualizing possible effects to be materialized within shorter periods, and in view also of the temptation to base empirical studies on current quarterly or annual statistics, the large family of multipliers must grow still larger in order to include ratios which refer to the values of incomes attained within shorter intervals.[1]

SOME VARIATIONS OF MODEL VIII

Variations of the propensities to save and to import reveal interesting properties of Model VIII, some of which are analogous to those of Model IV but others rather different.

Let us compare Tables VIII-a and VIII-b. While the propensities in country A are the same in these two cases, the propensities in B were doubled. In Table VIII-a, $s_B = .1$,

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analyzed, is now regarded as multiplicand; hence it no longer affects the multiplier.

[1] The list on page 120 gives a collection of multipliers for the values of Table VIII-a.

$m_B = .15$; in Table VIII-b, $s_B = .2$, $m_B = .3$. The effects of this doubling are concentrated on country B: the income increment in B is only half that of the former case. But there are no effects on the eventual income increment of A; and A's export surplus, which starts with \$30.00 now as against the \$15.00 before, ends up with the identical value of \$37.50. The suggestion is (as it was in Model IV) that proportional

	COUNTRY A: S=0.2 M=0.3								COUNTRY B: S=0.2 M=0.3							
					Foreign Balance			Income from 2+5				K=3.125	Foreign Balance			Income from 10+13+15
		Spendable Income			=12	=-14				Spendable Income			=4	=-6		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Period	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For.-Ind ΔX	Net Δ(X-M)	Auton. ΔI	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For.-Ind ΔX	Net Δ(X-M)	Auton. ΔI	Current ΔY
1														_	100.00	100.00
2					30.00	30.00		30.00	100.00	50.00	20.00	30.00	–	-30.00	100.00	150.00
3	30.00	15.00	6.00	9.00	45.00	36.00		60.00	150.00	75.00	30.00	45.00	9.00	-36.00	100.00	184.00
4	60.00	30.00	12.00	18.00	55.20	37.20		85.20	184.00	92.00	36.80	55.20	18.00	-37.20	100.00	210.00
5	85.20	42.60	17.04	25.56	63.00	37.44		105.60	210.00	105.00	42.00	63.00	25.56	-37.44	100.00	230.56
6	105.60	52.80	21.12	31.68	69.17	37.49		121.97	230.56	115.28	46.11	69.17	31.68	-37.49	100.00	246.96
7	121.97	60.99	24.39	36.59	74.09	37.50		135.08	246.96	123.48	49.39	74.09	36.59	-37.50	100.00	260.07
8	135.08	67.54	27.02	40.52	78.02	37.50		145.56	260.07	130.04	52.01	78.02	40.52	-37.50	100.00	270.56
9	145.56	72.78	29.11	43.67	81.17	37.50		153.95	270.56	135.28	54.11	81.17	43.67	-37.50	100.00	278.95
10	153.95	76.97	30.79	46.19	83.69	37.50		160.66	278.95	139.47	55.79	83.69	46.19	-37.50	100.00	285.66
11	160.66	80.33	32.13	48.20	85.70	37.50		166.03	285.66	142.83	57.13	85.70	48.20	-37.50	100.00	291.03
12	166.03	83.01	33.21	49.81	87.31	37.50		170.32	291.03	145.51	58.21	87.31	49.81	-37.50	100.00	295.32
∞	187.50	93.75	37.50	56.25	93.75	37.50		187.50	312.50	156.25	62.50	93.75	56.25	-37.50	100.00	312.50

TABLE VIII-b.—Investments increased in B.   Foreign-induced exports of A. Induced saving and imports.

changes in the marginal propensities of country B do not change the end-effects upon country A.

It is interesting to contrast in Table VIII-b the development of the income increment with that of the export surplus of country A. The income increment reaches the ten-per cent-neighborhood of equilibrium in the 12th period, whereas the export surplus arrives in its neighborhood zone as early as in the 3rd period, that is in the period immediately after the first increase in export was obtained.

If, as in case VIII-b, the marginal propensities to save are the same in the two countries, the sum of the incomes generated in both countries must be equal to the autonomous investment multiplied by the reciprocal of the marginal propensity to save.   How the aggregate income increment is distributed over the two countries depends then upon the marginal propensities to import.   For example, the marginal

	COUNTRY A: $S=0.2$ $M=0.3$							COUNTRY B: $S=0.2$ $M=0.2$								
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Period					Foreign Balance $=12$ $=-14$			Income from 2+5	$K=3.5714$				Foreign Balance $=4$ $=-6$			Income from 10+13+15
	Spendable Income								Spendable Income							
	Past ΔY	Current ΔC	Induced ΔS	Home Ind ΔM	For-Ind ΔX	Net Δ(X·M)	Auton. ΔI	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home Ind ΔM	For-Ind ΔX	Net Δ(X·M)	Auton. ΔI	Current ΔY
1															100.00	100.00
2					20.00	20.00		20.00	100.00	60.00	20.00	20.00	—	-20.00	100.00	160.00
3	20.00	10.00	4.00	6.00	32.00	26.00		42.00	160.00	96.00	32.00	32.00	6.00	-26.00	100.00	202.00
4	42.00	21.00	8.40	12.60	40.40	27.80		61.40	202.00	121.20	40.40	40.40	12.60	-27.80	100.00	233.80
5	61.40	30.70	12.28	18.42	46.76	28.34		77.46	233.80	140.28	46.76	46.76	18.42	-28.34	100.00	258.70
6	77.46	38.73	15.49	23.24	51.74	28.50		90.47	258.70	155.22	51.74	51.74	23.24	-28.50	100.00	278.46
7	90.47	45.24	18.09	27.14	55.69	28.55		100.93	278.46	167.08	55.69	55.69	27.14	-28.55	100.00	294.22
8	100.93	50.46	20.19	30.28	58.84	28.56		109.30	294.22	176.54	58.84	58.84	30.28	-28.56	100.00	306.82
9	109.30	54.65	21.86	32.79	61.36	28.57		116.01	306.82	184.10	61.36	61.36	32.79	-28.57	100.00	316.89
10	116.01	58.00	23.20	34.81	63.38	28.57		121.38	316.89	190.13	63.38	63.38	34.81	-28.57	100.00	324.94
11	121.38	60.69	24.28	36.41	64.98	28.57		125.67	324.94	194.97	64.99	64.98	36.41	-28.57	100.00	331.38
12	125.67	62.84	25.13	37.70	66.27	28.57		129.11	331.38	198.83	66.28	66.27	37.70	-28.57	100.00	336.53
∞	142.86	71.43	28.57	42.86	71.43	28.57		142.86	357.14	214.28	71.43	71.43	42.86	-28.57	100.00	357.14

TABLE VIII-c.—Investments increased in B.   Foreign-induced exports of A. Induced saving and imports.

propensities to save are .2 (or ⅕) in country *A* as well as in country *B*, both in Tables VIII-b and VIII-c.   The aggregate increment in incomes in equilibrium is $500.00 in each case. In case VIII-b the marginal propensities to import are .3 in both countries; country *A* gets $187.50 of the income increase, leaving $312.50 to country *B*, whose home investment has started the whole expansion.   In case VIII-c the marginal propensity to import is .3 in *A*, but only .2 in *B*.   Less of the

income expansion is transmitted from B to A in this case; A gets only \$142.86, B retains \$357.14.

A comparison of Tables VIII-c and VIII-d shows a rather startling coincidence (quite in line, however, with one observed in the analogous cases of Model IV). In case VIII-c, with $s_B = .2$ and $m_B = .2$, B's final income increment is \$357.14.

	COUNTRY A: S=0.2 M=0.3								COUNTRY B: S=0.1 M=0.45							
				Foreign Balance				Income from 2+5	K=3.5714			Foreign Balance				Income from 10+13+15
Period	Spendable Income			=12	=-14				Spendable Income			=4	=-6			
	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Net Δ(X-M)	Auton. ΔI	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Net Δ(X-M)	Auton. ΔI	Current ΔY
1															100.00	100.00
2					45.00	45.00		45.00	100.00	45.00	10.00	45.00	–	-45.00	100.00	145.00
3	45.00	22.50	9.00	13.50	65.25	51.75		87.75	145.00	65.25	14.50	65.25	13.50	-51.75	100.00	178.75
4	87.75	43.87	17.55	26.33	80.44	54.11		124.31	178.75	80.44	17.87	80.44	26.33	-54.11	100.00	206.77
5	124.31	62.16	24.86	37.29	93.05	55.76		155.21	206.77	93.05	20.67	93.05	37.29	-55.76	100.00	230.30
6	155.21	77.61	31.04	46.56	103.65	57.09		181.26	230.34	103.65	23.04	103.65	46.56	-57.09	100.00	250.21
7	181.26	90.63	36.25	54.38	112.59	58.21		203.22	250.21	112.59	25.03	112.59	54.38	-58.21	100.00	266.97
8	203.22	101.61	40.64	60.97	120.14	59.17		221.75	266.97	120.14	26.69	120.14	60.97	-59.17	100.00	281.11
9	221.75	110.87	44.35	66.53	126.50	59.97		237.37	281.11	126.50	28.11	126.50	66.53	-59.97	100.00	293.03
10	237.37	118.69	47.47	71.21	131.87	60.66		250.56	293.03	131.86	29.30	131.87	71.21	-60.66	100.00	303.07
11	250.56	125.28	50.11	75.17	136.38	61.21		261.66	303.07	136.38	30.31	136.38	75.17	-61.21	100.00	311.55
12	261.66	130.83	52.33	78.50	140.20	61.70		271.03	311.55	140.20	31.15	140.20	78.50	-61.70	100.00	318.70
13	271.03	135.51	54.21	81.31	143.42	62.11		278.93	318.70	143.41	31.87	143.42	81.31	-62.11	100.00	324.72
14	278.93	139.46	55.79	83.68	146.13	62.45		285.59	324.72	146.12	32.47	146.13	83.68	-62.45	100.00	329.80
15	285.59	142.79	57.12	85.68	148.41	62.73		291.20	329.80	148.41	32.98	148.41	85.68	-62.73	100.00	334.09
∞	321.43	160.71	64.29	96.43	160.71	64.28		321.43	357.14	160.71	35.72	160.71	96.43	-64.28	100.00	357.14

TABLE VIII-d.—Investments increased in B. Foreign-induced exports of A. Induced saving and imports.

In case VIII-d, with the propensities in country A unchanged and the propensities in country B drastically changed to $s_B = .1$ and $m_B = .45$, B's final income increment is exactly the same: \$357.14. The income increment of A, however, jumps from \$142.86 in case VIII-c to \$321.43 in case VIII-d. B's lower saving propensity, obviously, causes the composite income expansion to be so much bigger; and its higher import propensity causes all of this gain to be transferred to A.

No wavelike movements in the sequences of income changes, such as were found in Model IV, are observed in any of the

TIME SERIES OF TABLE VIII-d

($s_a = .2$, $m_a = .3$, $s_b = .1$, $m_b = .45$)
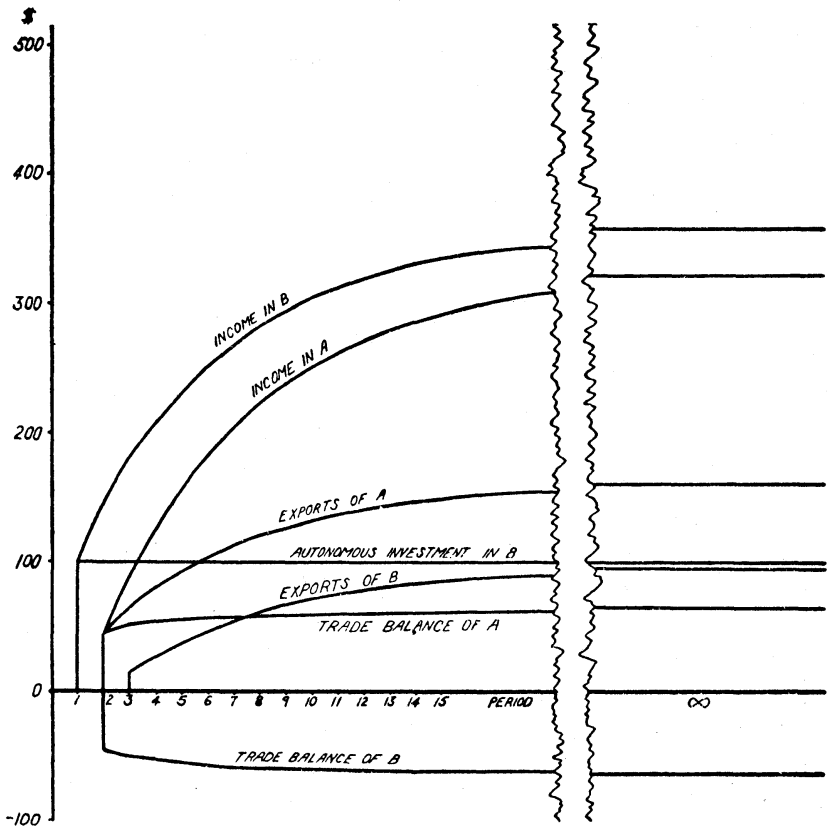
FIG. 5.—Time series of Table VIII-d.

tables for Model VIII. And, of course, the income changes are now positive in both countries. Figure 5 reproduces graphically the time series computed in Table VIII-d.

THE INVESTMENT MULTIPLIER WITH FOREIGN REPERCUSSIONS

While the figures in Tables VIII-a–d, showing gradual adjustments from period to period, were all computed as links of geometric progressions from the magnitudes of the immediately preceding incomes, the final equilibrium values were calculated with the help of multiplier formulas. The derivation of these formulas must now be shown.

Income in country A must grow until the new export surplus is matched by induced saving. Hence, changes in home investment being excluded, the equilibrium condition is—omitting the Δ signs as before—

$$(35) \qquad s_A Y_A = m_B Y_B - m_A Y_A.$$

From this it follows that

$$(35a) \qquad Y_A = m_B Y_B \frac{1}{s_A + m_A}.$$

It may be noted in passing that the value of exports, $m_B Y_B$, is here multiplied by the original, simple multiplier, for which we have not had any use since Model II, because it did not take account of foreign repercussions. This simple multiplier was not suitable for autonomous changes in foreign trade. Now it comes again into use for finding the income created by foreign-induced changes in foreign trade. It works here because these changes are already a "foreign repercussion," cast this time in the role of multiplicand.

For country B we may state that its income increment is the combined result of home investment, induced consumption and exports. In equilibrium, when received income and current income have become equal, the following equation must hold:

$$(36) \qquad Y_B = I_B + c_B Y_B + m_A Y_A.$$

Substituting (35a) in (36) we obtain in a few simple steps[1]

$$(37b) \qquad Y_B = I_B \frac{s_A + m_A}{(s_A + m_A)(s_B + m_B) - m_A m_B}.$$

This formula shows the home-investment multiplier of country B with all repercussions from country A. The multiplier can be written in the form given in (37b) or in any of these equivalent forms:

$$(37c) \qquad k_B = \frac{s_A + m_A}{s_A s_B + s_A m_B + s_B m_A} = \frac{1 + \dfrac{m_A}{s_A}}{s_B + m_B + s_B \dfrac{m_A}{s_A}}$$

$$= \frac{1 + \dfrac{m_A}{s_A}}{m_B + s_B \left(1 + \dfrac{m_A}{s_A}\right)}.$$

If we compare this investment multiplier with the one customarily in use, $\dfrac{1}{1 - c}$, we find that something new has been added to numerator and denominator. The numerator has been enlarged by $\dfrac{m_A}{s_A}$ and the denominator by $s_B \dfrac{m_A}{s_A}$. The addition to the numerator takes account of the fact that the

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[1] Equation (36) may be written as

$$(36a) \qquad Y_B(1 - c_B) = I_B + m_A Y_A.$$

Writing $(s_B + m_B)$ for $(1 - c_B)$ and substituting (35a), we get

$$(37) \qquad Y_B(s_B + m_B) = I_B + \frac{m_A m_B}{s_A + m_A} Y_B,$$

and

$$(37a) \qquad Y_B \left[(s_B + m_B) - \frac{m_A m_B}{s_A + m_A} \right] = I_B.$$

From this follows (37b) above.

transmission of the income expansion to the foreign country (A, in this case) brings back a repercussion in the form of an export increase, which reinforces the income expansion engendered by the increased investment.[1] The addition to the denominator is of much smaller weight; it takes care of the "repercussion of the repercussion," so to speak, allowing for the fact that the exports gained must reduce the foreign income expansion which is generated by the home-induced imports.

THE MULTIPLIERS COMPARED

We have found so far three different multipliers at work in this simple world of only two countries. The autonomous foreign-trade multiplier is the smallest of the three; next in size comes the foreign-induced-trade multiplier; the largest is the home-investment multiplier.

Replacing for once the A and B notations for the two countries by the suffixes d and f for domestic and foreign, respectively, we list the three multipliers here for convenient comparison:

Autonomous change in export or import:
$$k = \frac{1}{s_d + m_d + s_d \dfrac{m_f}{s_f}}.$$

Foreign-induced change in export:
$$k = \frac{1}{s_d + m_d}.$$

Autonomous change in home investment:
$$k = \frac{1 + \dfrac{m_f}{s_f}}{s_d + m_d + s_d \dfrac{m_f}{s_f}}.$$

We shall find in Chapter IX that still more formulas are needed to describe the effects upon incomes if simultaneous

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[1] The greater the m_A, the greater these exports; the greater the s_A, the smaller the income expansion in A and, therefore, the smaller the exports of B.

changes in home investments take place in both countries. Such changes are likely to occur in opposite directions under the impact of international capital movements; and this will be our main theme in the next chapter. But the analysis of Chapter X will be applicable also to simultaneous changes in investment in general, no matter what causes them and whether they are in the opposite or the same direction.

Chapter VIII

INTERNATIONAL CAPITAL MOVEMENTS: ACCOMMODATING, SPONTANEOUS AND NET. TRADE BALANCE AND CAPITAL FLOW: CAUSE OR EFFECT. EFFECTS ON PRIMARY DISBURSEMENTS

INTERNATIONAL CAPITAL MOVEMENTS

At several places in this book references to the foreign balance or to "induced foreign lending" were made, but a detailed discussion of the capital items in the international balance of payments has been postponed up to now. The analysis of changes in capital items is much more complicated than the analysis of current-item changes in the balance of payments. The mere mechanisms as such are not easy to understand; the problems of causation are perplexing.

If the handy pair of adjectives, autonomous and induced, is applied to international transactions on capital account, that is, to changes in the balance of indebtedness, one should note that the meaning of these adjectives cannot here be the same as in connection with transactions on current account. We had best define autonomous capital movements as those transactions on capital account which are not in response to any other change in the balance of payments. For reasons which will become clear only later, we shall want to know in the case of autonomous capital movements where the initiative for the transaction originates. That is to say, we shall have to ask whether an autonomous capital export from *A* to

B was spontaneous lending on the part of *A* capitalists or spontaneous borrowing on the part of *B* borrowers; only in rare cases is the initiative divided and the transaction based on simultaneous and deliberate decisions of both parties. With regard to the "induced" capital transactions we must bear in mind that the "induced" does not have here the specific meaning of "directly induced by a change in income" that it had in connection with imports.

There is a significant difference between items on capital account and items on current account, which must be brought out before we proceed. Many "entries" in the international balance of payments under the heading of capital movements are merely the reverse side, the rear aspect, so to speak, of a transaction which is entered as a separate item on the other side of the ledger. To ascribe causal significance (such as affecting foreign-exchange markets or money circulation or income flow) to both entries would in many cases be erroneous. To bring out this aspect, it is convenient to use the adjective "induced" for those capital transactions which are merely "responses" rather than "forces" in the markets of foreign and domestic funds.[1]

INDUCED OR "ACCOMMODATING" CAPITAL EXPORT

If, for example, an exporter in country *A* sells *B*-funds, which he receives in payment for his increased exports, to the

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[1] Ragnar Nurkse, *Internationale Kapitalbewegungen*, Vienna, 1935, p. 225, defines "induced capital movements" as those which result from changes in other items in the balance of payments. Carl Iversen, *Aspects of the Theory of International Capital Movements*, London, 1935, pp. 453, 465, *et. al.*, has obviously the same things in mind when he speaks of "short-term equalizing capital movements." The "equilibrating capital movements" in the terminology of Charles Poor Kindleberger, *International Short-term Capital Movements*, New York, 1937, p. 7, are somewhat less inclusive. He defines them as "those resulting directly from changes in other items in the balance of payments," which leaves those resulting less directly as separate categories. See below, footnote 1, on p. 135.

banking system of A, and the banking system of A decides to hold the B-balances, the records of the international balance of payments will show the merchandise export on the credit side and the foreign lending (which is implied in the banking system's increased holdings of claims against foreigners) on the debit side.[1] This foreign lending counter-balances the merchandise export in a bookkeeping sense but not in the sense of neutralizing its effects.[2] The statement that the export creates income is true only because somebody, e.g. the banking system, is willing to acquire foreign claims (or gold) and to release the domestic funds needed to finance the production of the export goods. Thus, this foreign lending (or gold purchase) must not be considered as a separate force, but rather as a response to, or perhaps an integral part of, the force exerted by the increase in exports.

The willingness on the part of banks or foreign-exchange dealers to acquire and hold foreign balances is usually dependent on the existence of a system of organized exchange-rate stabilization, such as the gold standard in one of its several

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[1] Increased holdings of deposits in foreign banks, together with increased holdings of foreign bills of exchange or other short-term claims against foreigners, are listed among the "short-term capital exports" (short-term foreign lending), that is, as debit entries, in the international balance of payments. It might be preferable to separate *explicit* foreign lending, such as the holding of additional foreign bills of exchange, from the *implicit* foreign lending which consists merely in holding larger deposits in foreign banks. Professor Viner, for instance, does not include changes in foreign bank balances when he speaks of foreign lending or borrowing, or even of "net" foreign lending or borrowing. (Cf. *op. cit.* pp. 423–25.) Most contemporary writers, however, follow the procedure of Lord Keynes in that all increments in balances held abroad are included in "foreign lending." (See J. M. Keynes, *A Treatise on Money*, London, 1930, Vol. I, pp. 131–32 and 161–166.)

[2] This explains two of the various meanings of "balance of payments." In the one sense, the balance of payments must always "balance," credit and debit entries can never be unequal. In the other sense, the balance of payments is called "favorable" if a credit item, though counter-balanced by a debit item, is not neutralized in its effects on exchange rates, circulation or income flow.

forms or the operation of an exchange stabilization fund.[1] The assumption of stable foreign-exchange rates—an assumption maintained all through our argument—implies the existence of an organization (central bank, stabilization fund, or a group of leading commercial banks) which is always prepared to buy any amounts offered, or to sell any amounts demanded, of foreign balances (or gold)—possibly at price pairs with only modest spreads between buying and selling quotations. By increasing its holdings of foreign claims and balances the banking system engages in foreign lending ("capital export") and, simultaneously (except in the case of deliberate offsetting operations), in an expansion of the domestic circulation. *Vice versa*, by reducing its holdings of foreign balances the banking system performs a "capital import" and (save for offsetting) a contraction of the domestic circulation.

If nobody were willing to increase his holdings of foreign balances (or gold), foreign-exchange rates could not remain stable and no income would be created through exports. Exporters would have to dispose of foreign funds to those who have immediate use for them, that is, importers. Increased exports would merely lead to a reduction of the price of foreign money low enough to induce an importer to use it for an increase in imports. Hence, an exchange-rate-induced increase in imports would offset immediately the autonomous increase in exports.[2]

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[1] For a brief discussion of various systems of organized exchange-rate pegging, ranging from uncoordinated actions of foreign-exchange speculators to the most dignified forms of the gold standard, see my article on "The theory of foreign exchanges," Part II, *Economica*, Vol. 7 (New Series) 1940, pp. 37–39.

[2] If the expression "favorable balance of payments" is used in the sense of a credit balance on current account (i.e. a net export surplus in merchandise and services) the situation referred to in the text would not constitute a "favorable balance of payments." If this ambiguous expression is used, however, to describe situations on the foreign-exchange market which involve, in a system of pegged foreign-exchange rates, an increase in the banks' or fund's holdings of foreign balances and gold; or, in a system of

The statement that an increase in exports or decrease in imports creates income presupposes, therefore, the existence of some degree of exchange-rate pegging; it implies the preparedness to accommodate the exporters who offer foreign claims or balances by taking these assets off their hands and holding them for the time being. This "induced" foreign lending may perhaps be characterized as "accommodating" lending. Behind this foreign lending is *not a spontaneous demand for foreign claims or securities* but rather a latent preparedness to accommodate those who come to dispose of foreign balances. In rare cases, and for a short while only, the exporters themselves may hold the foreign claims and finance the exports (the production of the export articles) out of idle funds (dishoarding) or by borrowing from banks. More often, however, speculators and, most often, the banking system (including exchange stabilization funds) will be making the accommodating foreign loans.

"Accommodating" should not mean here that it is done out of sheer kindness or as a service to customers. The "accommodating" purchases or sales of foreign bills and balances are profitable for speculators, dealers, banks and even monetary authorities. All except the monetary authorities engage in these transactions strictly for business reasons. We may distinguish three types of "accommodating" foreign lending according to the underlying motives. (*a*) Banks or capitalists buy the foreign balances in order to hold foreign interest-bearing claims (or securities). They are attracted by a differential between foreign and domestic interest rates (or relative yields), a differential which is created by or connected with the

flexible foreign-exchange rates, a decline in the price of foreign exchange, then the "finding" is different. The situation referred to in the text, where imports and exports balance completely, would then still be considered as a favorable balance of payments, because the export was the autonomous change and the import only induced by a reduction in the rates of foreign exchange.

increased supply of foreign bills or balances.   (b) Banks or
foreign-exchange dealers buy the foreign balances in order to
hold them until later when foreign-exchange rates are expected
to be a few points higher (i.e., near the upper of the stable pair
of prices).   They can anticipate this price differential because
of a slight downward pressure upon the current buying rates,
which, at the time, is being exerted by the increased supply of
foreign balances.   (Motives (a) and (b) are often working
together and, in the case of bills, are really unseparable.[1])
(c) Exchange stabilization funds or central banks buy the
foreign balances in order to prevent foreign-exchange rates
from falling.   They may either hold the foreign balances
until the market turns or convert them into gold.

SPONTANEOUS CAPITAL EXPORT

An entirely different matter is spontaneous foreign lending.
*Here it is the demand for foreign balances and securities which under-
goes a change* and calls for equilibrating adjustments.   Now,
in all systems which provide for some sort of stability of foreign-
exchange rates, a spontaneous change (positive or negative) in
foreign lending, that is, a spontaneous export or import of
capital, is likely to be "met" by accommodating changes of
opposite direction.[2]   An increased demand for holding foreign

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[1] See Viner, *op. cit.* p. 404.   Kindleberger, *op. cit.* pp. 8–9, calls movements
motivated by (a) "income short-term capital movements" and those moti-
vated by (b) "speculative short-term capital movements."   But these two
of Kindleberger's categories include more than our "induced" or "accom-
modating" capital movements; for, obviously, expectations concerning
interest or exchange-rate differentials may be due also to other things than
changes in the balance of payments.   Only to the extent as the interest (or
security price) and exchange-rate differentials are caused by other changes
in the balance of payments, are Kindleberger's "income short-term capital
movements" and "speculative short-term capital movements" part of what
we call here "induced" (or "accommodating") capital movements.

[2] Cf. Carl Iverson, *op. cit.* p. 465: " . . . the long-term real capital
movement . . . is temporarily offset by a short-term equalizing capital
transfer in the opposite direction."

assets is met by a latent supply, in that speculators or the banking system stand ready to part with the foreign balances which are newly demanded; *vice versa*, a decreased demand for holding foreign assets (i.e., a supply of foreign assets) is met by a latent demand for foreign balances, in that speculators or the banking system are prepared to acquire and hold the foreign balances which are offered. But it would be wrong to suppose that this counter-balancing of spontaneous changes by accommodating changes in foreign lending will as a rule neutralize the former ones with respect to their effects on circulation and income flow. In certain cases the accommodating capital movements do neutralize the spontaneous ones, but in other cases they do not.

Scrutiny of this problem will throw light upon an old controversial issue, an issue which bears some resemblance to the question of which came first, the chicken or the egg. The question of whether capital movements lead the trade balances, or trade balances direct the capital movements, has given rise to much discussion and it was somewhat bewildering that experts should arrive at opposite answers. The classical view was that capital movements were the cause and trade balances the effect.[1] The opposite tenet is that the trade balance is the cause and capital movements the effect.[2] And

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[1] For references to the writings of Hume, Thornton, Wheatley, Ricardo, Longfield, Torrens, Joplin, J. S. Mill, Cairnes, Bastable and Nicholson, see Viner, *op. cit.*, pp. 292–304. Among modern representatives of this view are Taussig, Wicksell, von Mises, Cassel, Angell, Ohlin, Iversen and many others. With some qualifications Professor Haberler may also be counted among the adherents of this view. Professor Angell, commenting on the case of Canada from 1900–1913, stated that the change in the balance of trade followed the capital movements with a time lag "that runs up to a year in length." (James W. Angell, *The Theory of International Prices*, Cambridge, Mass., 1926, p. 411.)

[2] This view was more prevalent first in Germany. Today, however, the outstanding representative of this theory is Lord Keynes. And, hence, it is now very widely held everywhere. Keynes opposed the theory of the adjustment of trade balances to capital movements on the ground that

there is also the compromise view that "there is no apparent *a priori* reason why the dependence should not be as much in one direction as the other."[1] We shall see that certain patterns of thinking seem to enforce a one-way dependence only, to the exclusion of any possibility of "the other" dependence, and yet, we shall find that these apparently contradictory patterns of thinking are reasonably acceptable and reconcilable.

Now, returning to the question from which we departed, what are the decisive points in the problem of the neutralization of spontaneous capital movements? Reasoning in terms of a spontaneous change in the domestic demand for foreign assets, we find that everything will depend on whether this change is at the expense (or in favor) of (*a*) idle funds, (*b*) bank debts, (*c*) domestic investment or (*d*) consumption. In the first two of these cases the spontaneous change in foreign lending may in fact not merely be balanced but also neutralized by the opposite accommodating change in foreign lending. In the last two cases, however, the balancing is not neutralizing. More concrete language may help to make this clear.

Let us assume a spontaneous increase in foreign lending: individuals or firms wish to acquire foreign claims or foreign securities. They buy them and pay for them (*a*) by reducing their liquid balances with domestic banks, or (*b*) by increasing

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foreign trade was not easily adjustable; one must not apply, he said, "the theory of liquids to what is, if not a solid, at least a sticky mass with strong internal resistances." And he states: "Historically, the volume of foreign investment has tended, I think, to adjust itself—at least to a certain extent—to the balance of trade, rather than the other way round, the former being the sensitive and the latter the insensitive factor." (J. M. Keynes, "The German transfer problem," *Economic Journal*, Vol. 39, 1929, p. 6.)

[1] Viner, *op. cit.*, p. 364. But Professor Viner continues (p. 365): "Examination of such data as are readily available strongly confirms, however, the orthodox doctrine . . . that major long-term capital movements have . . . mainly been 'disturbing' rather than 'equilibrating' in nature." Disturbing means here the same as autonomous, or being the cause; equilibrating means the same as induced, or being the effect.

their debts to domestic banks.   (The situation is no different if they finance themselves by selling domestic securities which are bought by the banks or by somebody who thereby reduces his liquid balances or increases his bank debts.)   In case (a) the banking system sells foreign balances to the individuals and firms who demand them, and finds its deposit liabilities reduced through the withdrawal (cancellation) of funds which have been inactive anyway.   (Expressed in the widely used symbols of Irving Fisher's equation of exchange, M' is reduced, V' increased, $M'V'$ unchanged.)   In case (b) the banking system, as in (a), gives foreign funds to those who demand them, but, this time, not against a reduction in deposits but against an increase in credit outstanding.   (Circulation, volume or velocity, is not affected at all.)   In both cases, in (a) as well as in (b), the domestic money flow is unaltered.[1]

While, customarily, one still speaks of an "unfavorable balance of payments," in neither of these cases does a change in foreign-exchange rates (beyond the few points compatible with "stability") or in the domestic money flow take place; all shocks are absorbed by the banking system.   But the same preparedness of the banking system will not prevent an effect upon the income flow if the adverse balance of payments is associated, as it would be in cases (c) and (d) respectively, with changes in domestic investment or consumption.   The spontaneous increase in demand for foreign assets—whether it is motivated by especially attractive yield expectations or whether it is a flight of capital[2]—may reduce domestic invest-

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[1] Of course, the story is not complete.   The switch in foreign balances or other assets may have effects on investment or consumption expenditures abroad; and repercussions through foreign trade—foreign-induced exports—may then be felt at home.

[2] I have dealt with the poblems of capital flight in a German article, "Die Theorie der Kapitalflucht," *Weltwirtschaftliches Archiv*, Vol. 36, 1932, pp. 512–529.

ment and consumption, either in that the buyers of the foreign assets themselves cut down their other disbursements or that they raise the needed funds in such a way as to cause others to reduce their investment and consumption expenditures.[1] Thus, in cases (c) and (d), the balancing of the spontaneous capital exports by accommodating capital imports does not neutralize the former; primary disbursements are reduced and the income flow will ebb.

NET CAPITAL EXPORT

Now it is a matter of taste how one chooses to treat the further adjustments from this point on. One may prefer to start with a fresh breath and regard the reductions in home investment and consumption as autonomous changes which, through a lowering of incomes, result in an induced fall in imports, hence, in an export surplus. Or one may prefer not to lose sight of the autonomous foreign lending from which things have started and which may have been associated not only with reduced investment and consumption at home but also with increased investment and consumption abroad. But in any case one may insist that up to this point there has not yet occurred any *net* lending; for, we must remember, the spontaneous increase in foreign lending was balanced by an accommodating decrease in foreign lending when the banking system gave up foreign balances out of its own holdings (or possibly out of new borrowings abroad which it arranged for *ad hoc*) and turned them over to those who had felt the spontaneous desire to make foreign loans or investments.[2] Only

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[1] For example, a failure to renew short-term loans to firms or consumers may force the debtors to reduce their expenditures drastically.

[2] As I mentioned before, Professor Viner does not include changes in the banks' foreign bank balances in his "net foreign lending" or "net foreign borrowing." Trying to interpret the Keynesian position, I am, of course, using the Keynesian concept of net foreign lending.

later, when reduced home investment and consumption are
cutting incomes in the "lending" country, and increased
home investment and consumption are raising incomes in the
"borrowing" country, will the "lending" country experience
both a home-induced fall in imports and a foreign-induced
rise in exports.   This favorable trade balance will, at least in
part, replace the foreign holdings with which the banking
system has first parted.   That is to say, the banking system,
which had accommodatingly performed negative foreign
lending as it gave its balances to those who spontaneously
performed positive foreign lending, will now acquire the
foreign balances which exporters (with their business increased)
cannot sell (at stable exchange rates) to importers (with their
business reduced); thus, the banking system will now accom-
modatingly engage in positive foreign lending.   This means
that it is, after all, the improved trade balance which, by
reversing the direction of *accommodating* capital exports, permits
the *spontaneous* capital exports, or a part of them, to become *net*
capital exports.

One can now understand why some writers prefer to place
all the emphasis on the adjustment of the net capital flow to
the trade balance: because this so-called adjustment is a cer-
tainty, whereas the adjustment of the trade balance to the
spontaneous capital exports is only a probability.   There is
nothing which assures equality between spontaneous capital
exports and the changes in investment and consumption
expenditures; nor is there any equality between the changes in
these expenditures and the improvement in the trade balance.
Therefore, important as the relationship may be for the
explanation of reality, the quantitative relationship between
spontaneous capital export and improved trade balance is
rather uncertain.   On the other hand, the net capital export
(together with a possible gold inflow) is necessarily equal to the

improvement of the trade balance. This equality follows directly from the definitions of the items involved; the statement cannot be wrong; but whether this statement is very useful in the explanation of actual phenomena is another matter.[1]

CAUSE OR EFFECT

We should not quarrel about tastes. One way of exposition, more nearly neo-classical in character, has the advantage of tracing the whole sequence of events from the autonomous movement of capital, via the reactions of the monetary system, to the trade balance. The other way has the advantage of reducing the whole network to the simplicity of the investment-and-consumption scheme. Since, as we have seen, spontaneous foreign lending becomes net foreign lending only if it affects, in at least one of the countries concerned, the disbursements for home investment and consumption, it is justifiable to start from there. But starting from there, that is from the changes in investment and consumption, one cannot help seeing all movements of capital only as induced ones, effected by the income-induced changes in the trade balance. These changes in the trade balance are simply those shown by the

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[1] The logical difference between the two statements about capital movements and trade balance should be noted. The one which says that the trade balance adjusts itself to the capital movements—read: *spontaneous* capital movements—, expresses a mere probability; this statement remains in the realm of *causality*. The other, which says that the capital movements —read: *net* capital movements—adjust themselves to the trade balance, expresses an absolute certainty; thus this statement belongs to the realm of *tautology*. Only if one were willing to change the second statement so that it would refer no longer to net capital flows but instead to certain types of capital flows, for instance long-term capital movements, would the statement change its logical character and become one about cause and effect; its probability value, however, would be definitely inferior to that of the opposite proposition. (Cf. Professor Viner's remark quoted in footnote 1, p. 137 above.)

theory of the home-investment multiplier. Expressed in a few catchwords, the train of thought is then the following:

Country A	Country B
Decrease in home investment and consumption	Increase in home investment and consumption
↓	↓
Decrease in incomes	Increase in incomes
↓	↓
Induced decrease in imports	Induced increase in imports
↓ ⟵	⟶ ↓
Export balance of trade	Import balance of trade
↓	↓
Net export of capital	Net import of capital
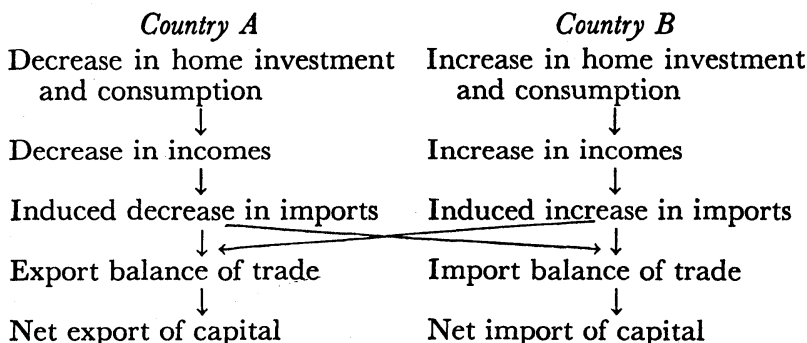
This may not look any longer like a theory of the effects of spontaneous capital exports—which apparently have taken place "before the curtain rose"—but it is a perfectly acceptable and convenient pattern of reasoning.

I believe that our discussion has disposed of some of the seemingly irreconcilable differences in this chicken-or-egg controversy between the "neo-classical" and "neo-mercantilist" schools of thought.   In both types of analysis the changes in disbursements for investment and consumption in one or both countries and the concomitant changes in incomes, are essential steps.   The change in the trade balance may be attributed with equal justification either to the changes in incomes or, tracing it back one or two steps further, to the spontaneous demand for foreign assets on the part of capitalists in the lending country (that is, to that change in demand from which changed disbursements and incomes have originated). The capital export may be detected with equal reasonableness either in the *spontaneous* demand for foreign assets or in the accommodating foreign lending which consists in acquiring the foreign balances derived from the export surplus and which accounts for the eventual *net* balance of increase over

decrease in the holdings of foreign claims. And, of course, if the words "capital movements" stand for *net* capital movements including all changes in foreign bank balances,[1] then one can always maintain and "prove" (with a reservation concerning gold flows) that capital movements "result" from the trade balance (or, more correctly, from the balance of current items). On the other hand, if the words "capital movements" stand for *long-term* capital movements, then one must say that all the alternative patterns of thinking make it perfectly plausible that the trade balance can adjust itself to a capital flow which precedes it. Whether the capital movement, in the long-term sense, will in reality more often lead or follow the trade balance, becomes again a real question. This question cannot be decided merely by implicit theorizing; but the evidence presented by careful investigators points to the more frequent priority of capital movements.[2]

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[1] This includes not only interbank deposits but also debit and credit balances of *A*-nationals in *B*-banks and of *B*-nationals in *A*-banks.

[2] Professor Viner, after a painstaking examination of the available data, takes Canada from 1900–1913 as an example of a case where foreign borrowing can be considered as the cause of the import surpluses. The evidence is fully convincing and whatever doubts were raised by critics referred only to the exact mechanisms by which the capital imports translated themselves eventually into commodity imports. The sequence of events was probably something like the following: Flotation of Canadian issues in London—Use of parts of the sterling proceeds to pay for interest obligations and for certain imports of investment goods—Sale of rest of the sterling balances to Canadian banks—Use of the Canadian bank funds (received in exchange) for investment expenditures in Canada—Expansion of Canadian incomes (and advances in relative prices)—Increased Canadian imports—Utilization of the foreign balances of Canadian banks for paying for the import surplus.

Nonetheless, it is possible to argue differently. As Professor Harold M. Somers has pointed out to me, one may attribute the fact that Canadian issues were floated in England rather than in Canada to an institutional factor, namely the excellence of the London capital market compared with the Canadian. If Canadian banks had initially acquired the new issues, the sequence would have been different, with the imports leading and the foreign borrowing trailing. For example: Flotation of new issues in Canada, acquired by Canadian banks—Use of the (thereby created)

TYPES OF AUTONOMOUS CAPITAL MOVEMENTS

The theory of the effects of autonomous capital movements upon trade balances and national incomes may be conveniently divided into two parts. First comes an analysis of the effects of the spontaneous capital transactions upon aggregate disbursements for home investment and consumption in the countries concerned. Second comes the analysis of the effects of changes in investment and consumption upon trade balances and incomes. The first was characterized above as the story that takes place before the curtain rises over the drama of the second analysis, a drama, incidentally, which can be produced with the technique of the home-investment multiplier (applied, of course, to at least two countries at the same time). Although the present study is chiefly concerned with the part that can be treated with the multiplier tech-

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Canadian bank funds for investment expenditures in Canada—Expansion of Canadian incomes (and advances in relative prices)—Increased Canadian imports—Demand for foreign balances to pay for the import surplus—Drain on Canadian bank reserves—In order to replenish their reserves Canadian banks sell in the London market the bonds out of their portfolio.

The end effects of both sequences are the same. In neither of the two cases could import surpluses continue for a long time were it not for the English purchases of Canadian bonds. Yet the initiative to sell the bonds in England is of a different character in the second sequence: it can be said in this case that the sales are forced upon the Canadians by their unfavorable trade balance. This time the foreign borrowing has not led but followed the import surplus.

Professor Viner does not rule out interpretations of this sort. He says himself, with reference to the Canadian experience: "From the data . . . it is possible to argue that at times at least the import surpluses resulted from original secondary expansion, and that the borrowings were engaged in to obtain the foreign funds necessary to liquidate trade balances already incurred and to restore reserves encroached upon in paying for past debit trade balances." (*Op. cit.*, p. 429.) In general "there was a variable time-lag between borrowings abroad and economic transfer, with the recorded, or long-term, borrowings usually but not always preceding the economic transfer chronologically." (*Op. cit.*, p. 431.)

Professor Viner mentions New Zealand after 1919 as an example of a case where a loss of exports can be regarded as the cause of borrowing from abroad. (*Op. cit.*, p. 365.)

nique, it seems appropriate to gain some insight in the "antecedent actions." How does the spontaneous capital flow affect primary disbursements in the countries concerned?

The answer will depend on several circumstances, usually described in terms of liquidity positions and investment opportunities, but it depends also to a considerable extent on the type of transactions which constitute the capital outflow or inflow. To analyze the "several circumstances" would involve almost the whole theory of investment; we cannot undertake such an enterprise here. But to get some idea about the influence which the nature of the international capital transactions may have upon the rate of domestic disbursements is a possible task. It calls for distinguishing among a few rather fundamental types of autonomous capital movements. Such an attempt to classify them according to their nature and the character and locus of the initiative from which they originate may be helpful, although it cannot be claimed that the principle of classification is sound for all purposes or that our list is complete in any respect. (The point of going into the question of initiative and spontaneity is pretty much the same as that of inquiring whether in a commodity market an increase in the quantity sold is due to an increase in demand or an increase in supply.)

Every *autonomous* capital export of one country is, by definition, an autonomous capital import of another country. (Autonomous, we repeat, does not mean here "not induced by any changes in income" but means instead "not in response to other changes in the international balance of payments.") But an autonomous capital movement need not be *spontaneous* on the part of both the lending and the borrowing country; that is to say, the initiative may be that of only one of the two parties. We shall find in our classification cases of strictly one-sided spontaneity, but also some of joint spontaneity, so that cases listed once for the capital exporting and again for

the capital importing country may be merely two aspects of an identical transaction. In such cases the supply of funds (demand for assets) by the exporting country and the demand for funds (supply of assets) by the importing country can both be regarded as increased.

The classification which is offered here is designed to maintain symmetry.[1] But one should bear in mind that most items have more than one possible opposite. One pair of opposites, for example, is "lending" and "borrowing"; but "lending" and "collecting" is another. It is the second type of opposites which underlies the symmetry design of our classification.

SPONTANEOUS CAPITAL EXPORTS

(1) Increased demand for holding foreign assets[2] involving *"direct investment" abroad* under the control of the capital exporter.

(2) Increased demand for holding foreign assets involving *"direct lending" to borrowers abroad* (with direct contact between lender and borrower but without "control").

(3) Increased demand for holding foreign assets resulting in *acquisition of old foreign securities* or other assets, without significant contacts with foreign sellers or debtors.

(4) Increased demand for holding foreign assets resulting in *subscriptions to new issues of foreign securities.*

(5) Increased demand for holding domestic assets held in foreign hands resulting in *"repatriation" of domestic securities* or other assets.

(6) Increased demand to reduce one's own debts to foreign creditors, effected in (premature) *debt repayments to foreigners.*

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[1] For the sake of symmetry, separations which suggested themselves for the list of spontaneous capital exports were made also in the list of spontaneous capital imports or *vice versa*, even if they might serve there no useful purpose.

[2] "Foreign Assets" include claims against foreigners, foreign securities (bonds and shares), rights and other sorts of property abroad.

(7) Fulfillment of foreign obligations (at maturity) and *unilateral payments to foreign countries,* such as indemnities, reparations, etc.

SPONTANEOUS CAPITAL IMPORTS

(8) Decreased demand for holding foreign assets involving *sale or liquidation of direct investment abroad.*

(9) Decreased demand for holding foreign assets involving recall or *non-renewal of direct loans to foreign debtors.*

(10) Decreased demand for holding foreign assets resulting in the supply and *realization of foreign securities* or other assets by domestic holders, without significant contacts with foreign buyers.

(11) Increased supply of domestic assets for sale in foreign markets, including *flotations of new issues abroad.*

(12) Increased supply of domestic assets to foreign investors, including *receipt of "direct investments" from abroad.*

(13) Increased demand for foreign loanable funds resulting in *direct borrowing from foreigners.*

(14) Collection of foreign obligations (at maturity) and receipts of *unilateral payments from foreign countries,* such as indemnities, reparations, etc.

The matter of "autonomous but not spontaneous" capital exports or imports can now be made clearer by reference to examples from these lists. The spontaneous capital export of type (3), taking the form of securities purchases by capitalists in the capital exporting country, is not likely to be spontaneous also from the point of view of the capital importing country. The sellers of the securities in the latter may not even know who acquires their former holdings. Their sale of securities, since it is actually a sale to foreigners, constitutes an autonomous long-term capital import, but it is not a spontaneous capital import,[1] that is, behind it is not any

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[1] And it is not an induced capital import either. On the contrary, the induced (accommodating) capital flow which will accompany the transac-

increase in the demand for funds.   An example of two-sided
initiative, joint spontaneity between the lending and the
borrowing countries, may be found in case (1) of the above
list, the counterpart of which, from the point of view of the
capital importing country, is type (12).   Likewise, types (2)
and (13) may be the two aspects of an identical transaction,
seen from the two countries in question.   And, surely, (7) and
(14) are merely two aspects of the same thing, because if
obligations are fulfilled at appointed dates the initiative must
be regarded as common to debtor and creditor.   It is proba-
bly safe to say that, in our times of "security capitalism,"
one-sided spontaneity is more frequent than joint spontaneity.

The chances that capital movements will affect the levels
of consumption and home investment in the capital exporting
and in the capital importing country are quite uneven in the
various types of movements.   In general, subject to qualifica-
tions, it seems that any such effects on domestic disbursements
are most likely to arise in the country in which a spontaneous
capital import takes place.   The case of "direct investments"
undertaken in a foreign country is the one most likely linked
up with increased disbursements in that country.   In most
other cases the shocks originating from the capital movements
may be partly (or even completely) absorbed by flexible
monetary mechanisms without much impact on primary
disbursements.

PRIMARY DISBURSEMENTS IN THE CAPITAL
EXPORTING COUNTRY

Looking first into possible effects upon primary *consumption*
expenditures, we begin with the unilateral payments [cases
(7) and (14)].   If it is government funds (e.g. indemnity

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tion will be a capital export from this country, because its banking system
will be building up reserves in foreign balances as it acquires the proceeds
from the securities sales.   This is not to deny that securities transactions
can be also induced (accommodating) as was pointed out before (p. 134).

payments) that are transferred abroad and if these funds are being raised through taxation borne by lower-income classes, restrictive effects on consumption are certain to be felt. But if the government raises the funds through borrowing, no primary restriction of domestic consumption need arise. (In the receiving country the effects will depend on the use that is made of the received funds. Their use for government expenditures does not yet assure *increased* government expenditures; the received funds may serve merely to continue an unchanged level of expenditures with a reduced level of domestic borrowings. If the received funds are used to compensate for tax reductions, increased consumption may result, but probably only if the taxes which are reduced were borne by lower-income groups.)

Effects on primary consumption expenditures in the case of private capital transactions are not likely. Perhaps if the "capital exporter" raises the funds he needs for the acquisition of foreign claims or securities by "calling in" domestic loans outstanding, an encroachment upon primary consumption is possible. If the spontaneous export of capital is so "urgent" —as it is likely to be only in the case of a flight of capital—that not only domestic loans are seriously restricted but also the capitalists' propensity to save rises sharply, then a substantial fall of primary consumption will be associated with the rise in the demand for foreign assets. (In the receiving country there is nothing which would produce a symmetrical increase in primary consumption expenditures.)

The qualifying word "primary" (by which statements about "consumption expenditures" have been modified here all the way through) is quite essential. For even if consumption is not directly affected by the inflow or outflow of capital, changes in investments in one or both of the countries concerned will, through changes in incomes, result in secondary (induced) increases or decreases of consumption expenditures.

That changes in domestic *investment* are associated with changes in the domestic demand for holding foreign assets or with increased borrowing or taxation for government payments abroad is probable but not certain.   If the acquisition of foreign assets for some reason appears more attractive, it may well take the place of home investment.   Thus, in all spontaneous capital export cases home investment may be encroached upon.   However, in times of easy money, a large part of the funds for the acquisition of foreign assets will probably come from liquid reserves and new bank credits; to the extent that this takes place without a stiffening of the interest rate, the level of current home investment will not be reduced.[1]   Liquidity reserves are not inexhaustible, of course; so one might assume that sooner or later, if the process has started from a saving-investment equilibrium and if the new demand for foreign assets is maintained, this demand must be satisfied at the expense of domestic investment.   But this is not necessary at all.   If, while the first installments of the spontaneous foreign lending are met out of liquidity reserves, investments begin to increase in the receiving country, expanding income in the latter will induce commodity purchases from the lending country.   And these new exports will do two things in the capital exporting country.   One is that the reserve position of the banking system, possibly weakened through the spontaneous capital outflow (or, that is, accommodating capital import in which they had to engage) is relieved through the foreign proceeds from the commodity exports.   The other is that the new exports generate income which results in induced saving.   And this increased saving can assist in feeding further installments of the spontaneous capital outflow without encroachment upon the given level of home investment.

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[1] We are faced here with our cases (*a*) and (*b*) of the four ways of financing foreign lending, referred to above on pp. 137–139.

PRIMARY DISBURSEMENTS IN THE CAPITAL IMPORTING COUNTRY

The likelihood that the capital movements of types (1)–(6), which are all spontaneous from the point of view of the lending country, actually will raise current investment in the receiving country is not equally great in all cases. The probability is greatest in type (1), the counterpart of which is listed as type (12). Surely this case of "direct investment" implies that the transferred funds are also disbursed dollar for dollar, since the investment is under the direct supervision of the capitalists of the lending country; the disbursement of the funds in the receiving country, except to the extent that machines are transferred in kind or other equipment is brought in from abroad, is almost part and parcel of the international transaction. A qualification may be needed concerning that part of the direct investment which goes into purchases of real property and other not newly constructed or produced assets, and thus does not represent additional primary disbursement. In the case of "direct foreign lending" and "direct foreign borrowing" [types (2) and (13)], when the initiative for the capital movement is as much on the side of the borrower as it is on the side of the lender, it would likewise be rather exceptional if the recipient did not make use of the borrowed funds. But we must not forget the possibility that the direct foreign borrowing may be merely a refunding transaction, which would release investible funds to the money market of the receiving country but would not directly contribute to current investment.

If interest rates were flexible, and if liquidity preferences (the demand for money balances for "speculative motives") were not too elastic, and if the marginal efficiencies of capital were sufficiently elastic, one might say that immediate investment by the first recipients of funds from abroad was not

essential at all. For, if the increased supply of loanable funds succeeded in lowering interest rates without much raising the inactive liquid balances, the use of large portions of the funds for real investment would be assured. (It would even "necessarily follow" from all these assumptions.) But the "ifs" are too many and too unlikely to be realized.

Thus, if the capital import is merely the reflexion of foreign demands for securities [type (3)] without any direct contacts between the capitalists of the "lending" country and the recipients of the funds in the "borrowing" country, the effects are indeed uncertain. Sellers of the securities which are acquired by the capitalists of the lending country may decide to hold liquid balances in the place of the sold assets and nobody may be induced to embark on real investment ventures. Or the banking system may absorb the entire shifts between types of foreign indebtedness with no more real change than an increased willingness to make loans (in the receiving country) if such were demanded. Hence, the autonomous capital movement may completely fail to call forth an increase of investment in the "borrowing" country.[1]

One way in which foreign purchases of securities may translate themselves into increased primary disbursements in the capital importing country is through reactions of the stock exchanges. There is a certain probability that the stock markets react favorably to the increased foreign purchases, and that higher stock prices call forth new issues for new real investment and, through the stimulus of capital gains, also

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[1] It may be well to remember that all mentioned capital movements are not yet *net* movements; the "borrowing" country is not yet net-borrowing. Hence, if no changes in investments are achieved, incomes and trade balances do not change either and all autonomous increases in the foreign indebtedness of the "borrowing" country must be continuously offset by accommodating decreases in its foreign indebtedness. For instance, while security holders sell out to foreigners and hold domestic bank balances, the banking system holds increased amounts of foreign balances ("outside reserve").

increased consumption.[1] Little can be said about the degree of probability or about the magnitudes involved. The resulting additional investment and consumption may be anything between zero and a multiple of the autonomous capital inflow; that they may be just equal to it is one among an infinite number of possibilities.

Little needs to be added concerning the cases where the securities sales are spontaneous in the capital importing country. The chance that spontaneous realizations of foreign assets [type (10)] will be associated with increased home investment is greater than in the cases of non-spontaneous securities sales, although it will still depend on the conditions of the money and capital markets. (But now it is the effects upon the disbursements in the capital exporting country which are all in the dark. Those, in the capital exporting country, who are induced to take over—repatriate—the securities which are spontaneously sold by holders in the capital importing country, will rarely do so at the expense of current home investment, though this would not be inconceivable.) In the case of new securities issues in foreign markets [type (11)] the association with increased home investment in the capital-importing country is fairly clear. (However, in the capital-exporting country the easy credit conditions which attract the new flotations are possibly the result of such a degree of liquidity that little or no inroads on home investment in the capital-exporting country need take place.)

REASONABLE ASSUMPTIONS

The foregoing account of the probable effects of autonomous capital movements upon primary disbursements was designed merely to permit a tentative and very general orientation and

[1] See my book *The Stock Market, Credit, and Capital Formation,* London and New York, 1940, pp. 107 ff., 159, 316 ff., *et al.*

to give an idea about the reasonableness of certain assumptions for the next step in our analysis. I am aware that I have neglected or unduly minimized the operations of various auxiliary mechanisms—money markets, securities markets, foreign-exchange markets, commodities markets—which can be instrumental in bringing about some closer adjustments between capital transactions and primary disbursements. But a more intensive study could be fruitful only if problems of *timing* were included, timing in the sense of the order in, and the lag with, which actions take place, and also in the sense of the phase of the business cycle during which these actions occur. This is far too big a problem for this monograph to tackle.

What the discussion has definitely taught us is that we cannot be satisfied with one "standard assumption." We shall have to work with all three types of assumptions: that primary disbursements in the paying country fall more than they rise in the receiving country; that they fall less in the paying country than they rise in the receiving country; and that they fall in the paying country by the same amount by which they rise in the receiving country. (The assumption that they remain unchanged in both countries requires no further analysis in this context, for obviously, in this case nothing changes except the positions of the banking systems.)

Instead of "primary disbursements" or "home investment and consumption" we shall often from now on, for the sake of economy of words, simply say "investment." And all these changes of "investment" will be called "autonomous" because they are not induced by income changes. It is understood that they may be induced by factors directly or indirectly linked with the autonomous movement of capital from country A to country B.

Chapter IX

THE NINTH AND TENTH MODELS: INVERSE CHANGES IN INVESTMENTS IN TWO COUNTRIES. THE INVESTMENT MULTIPLIER INVOLVING TWO COUNTRIES. THE TRANSFER PROBLEM. THE ELEVENTH MODEL: PARALLEL EXPANSION IN TWO COUNTRIES WITH BALANCED TRADE

MODEL IX: INVESTMENTS REDUCED IN A, INCREASED IN B; INDUCED IMPORTS AND EXPORTS, NO INDUCED SAVING

The effects of a simultaneous reduction in the current rate of home investment in country A and increase in the current rate of home investment in country B upon the trade balances between the countries and upon their income levels will be shown again with the help of simple models. As in the case of autonomous changes in foreign trade, we shall first exhibit a model which excludes induced saving. We may remember that, in the case of autonomous changes of trade, the exclusion of induced saving resulted in complete rectifications of the trade balances (Models I and III). An autonomous increase in exports of country A was followed, in Model I, by induced increases of imports and, in Model III, also by induced decreases of exports, until the trade balances were back at the

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zero level from which they had been pushed by the autonomous change. So it will be interesting to see what the absence of leakages and seepages through induced saving will do to the trade balances if autonomous changes occur in the rates of home investment in the two countries.

For the sequences in Table IX-a we choose equal and opposite changes in autonomous home investment—a reduction by $100 per period in A, an increase by $100 per period in B—, zero marginal propensities to save in both countries,

	COUNTRY A: S=0.0 M=0.3								COUNTRY B: S=0.0 M=0.4							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Period	K=1.4285			Foreign Balance				Income from 2+5+7	K=1.4285			Foreign Balance				Income from 10+13+15
	Spendable Income			=12	=-14				Spendable Income			=4	=-6			
	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Net Δ(X-M)	Auton. ΔI	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home Ind ΔM	For-Ind ΔX	Net Δ(X-M)	Auton ΔI	Current ΔY
1							-100.00	-100.00							+100.00	+100.00
2	-100.00	-70.00		-30.00	+40.00	+70.00	-100.00	-130.00	+100.00	+60.00		+40.00	-30.00	-70.00	+100.00	+130.00
3	-130.00	-91.00		-39.00	+52.00	+91.00	-100.00	-139.00	+130.00	+78.00		+52.00	-39.00	-91.00	+100.00	+139.00
4	-139.00	-97.30		-41.70	+55.60	+97.30	-100.00	-141.70	+139.00	+83.40		+55.60	-41.70	-97.30	+100.00	+141.70
5	-141.70	-99.19		-42.51	+56.68	+99.19	-100.00	-142.51	+141.70	+85.02		+56.68	-42.51	-99.19	+100.00	+142.51
∞	-142.85	-99.99		-42.86	+57.14	+100.00	-100.00	-142.85	+142.85	+85.71		+57.14	-42.86	-100.00	+100.00	+142.85

TABLE IX-a.—Investments reduced in A, increased in B by equal amount. Induced imports and exports. No saving.

but different marginal propensities to import—$m_A = .3$, $m_B = .4$. (Induced changes in home investment are again ruled out, as has been done consistently in this study. It is also ruled out, as before, that the autonomous changes in home investments affect the import demands directly. In other words, none of the $100 by which investments are increased in country B, are used directly for the importation of equipment or materials from A.)

Incomes, we see, are declining in A and increasing in B. Thus it is obvious that home-induced imports are declining and foreign-induced exports rising in A, and *vice versa* in B. With the export or import surpluses as the only offsets to the changed investment expenditures, incomes will stop falling or

rising, respectively, when the trade balances become equal (and opposite) to the changes in home investment. No later than in the third period we find the trade balances within 10 per cent of these values; and the changes of income reach the neighborhood zone of equilibrium already in the second period.

The ultimate levels of income show equal and opposite changes in the two countries, the decrement being 142.85 in A and the increment of the same size in B. That the marginal propensities to consume domestic products and services are different in the two countries does not prevent them from having the same multipliers. The reason is that the country which has the smaller marginal propensity to import and, thus, is less affected by home-induced changes in imports, is at the same time more heavily affected by foreign-induced changes in exports, and *vice versa*. The combined effects must be equal in the two countries. If the algebraic sum of the income changes were not zero—i.e., if the decrease in A and increase in B were not equal—there could be no equilibrium in a case with zero induced saving. We shall see presently why this is so.

Before leaving this case it should again be emphasized that here the autonomous changes in investment translated themselves completely into induced changes in the trade balances. If autonomous capital movements were the cause for the dollar-for-dollar changes in the rates of home investment, the adjustments of the trade balances to the capital movement would be perfect. Income effects alone, without the aid of changes in relative prices, would have accomplished the adjustment. Incomes, of course, remain at the lower level in A, and at the higher level in B, as long as the capital movements and the changed levels of home investment continue.

One might be tempted to credit the absence of induced saving always with particularly nice and smooth adjustments of

trade balances to capital movements. This would be a serious mistake, for as soon as the increase in the rate of investment in B is not exactly equal to the reduction of investment in A, trouble arises. With non-symmetrical changes in investment and without induced saving, sequences would fail to approach an equilibrium; they would what some writers like to call "explode," that is, they would move toward infinity. Such explosions are frightening only to economists who neglect the time element. To economists who employ a period analysis and give due regard to the lapse of time, the "explosions" would appear to be very gradual. Hence, they would not be explosions at all but merely movements into one direction which can be stopped only through factors omitted from considerations or through a change in the "data."

In Table IX-b we see such a case, where trade balances eventually find an equilibrium but national incomes find none. The changes in income go on and on, in a negative direction. Assumed is that the rate of current investment is reduced by \$100 in country A, but increased by only \$50 in country B. There is no induced saving either in A or in B; the marginal propensities to import are .3 in A, .2 in B.

Incomes in B, we observe, are first raised through the increased investment and induced consumption, but are soon pulled down through a severe loss of exports to country A. Incomes in A are shrinking because of the reduced level of primary investment and, thus, A's imports from B must shrink. The continuous deterioration of the situation in both countries begins to proceed after a while at constant absolute rates of change: incomes in A, from the 14th period on, fall by exactly \$20 per period, incomes in B by exactly \$30 per period. This impoverishment in the total amount of \$50 per period is easily explained by the fact that the money flow of the two countries combined is deprived every period of another \$50 through the \$100 reduction of investments in A which is half

offset by the $50 investments in *B*. Since the foreign-trade balances of the two countries combined must cancel out and there is no induced dissaving to offset the reduced rate of investment, the fall in incomes cannot be arrested. The distribution of the continuous losses between *A* and *B* ($20 : $30)

	COUNTRY A : S=0.0 M=0.3								COUNTRY B : S=0.0 M=0.2							
	K=∞			Foreign Balance				Income from 2+5+7	K=-∞			Foreign Balance				Income from 10+13+15
	Spendable Income			=12	=-14				Spendable Income			=4	=-6			
Period	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Net Δ(X-M)	Auton. ΔI	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Net Δ(X-M)	Auton. ΔI	Current ΔY
1							-100.00	-100.00							+50.00	+50.00
2	-100.00	-70.00		-30.00	+10.00	+40.00	-100.00	-160.00	+50.00	+40.00		+10.00	-30.00	-40.00	+50.00	+60.00
3	-160.00	-112.00		-48.00	+12.00	+60.00	-100.00	-200.00	+60.00	+48.00		+12.00	-48.00	-60.00	+50.00	+60.00
4	-200.00	-140.00		-60.00	+10.00	+70.00	-100.00	-230.00	+50.00	+40.00		+10.00	-60.00	-70.00	+50.00	+30.00
5	-230.00	-161.00		-69.00	+6.00	+75.00	-100.00	-255.00	+30.00	+24.00		+6.00	-69.00	-75.00	+50.00	+5.00
6	-255.00	-178.50		-76.50	+1.00	+77.50	-100.00	-277.50	+5.00	+4.00		+1.00	-76.50	-77.50	+50.00	-22.50
7	-277.50	-194.25		-83.25	-4.50	+78.75	-100.00	-298.75	-22.50	-18.00		-4.50	-83.25	-78.75	+50.00	-51.25
8	-298.75	-209.13		-89.62	-10.25	+79.37	-100.00	-319.38	-51.25	-41.00		-10.25	-89.62	-79.37	+50.00	-80.62
9	-319.38	-223.57		-95.81	-16.12	+79.69	-100.00	-339.69	-80.62	-64.50		-16.12	-95.81	-79.69	+50.00	-110.31
10	-339.69	-237.78		-101.91	-22.06	+79.85	-100.00	-359.84	-110.31	-88.25		-22.06	-101.91	-79.85	+50.00	-140.16
11	-359.84	-251.89		-107.95	-28.03	+79.92	-100.00	-379.92	-140.16	-112.13		-28.03	-107.95	-79.92	+50.00	-170.08
12	-379.92	-265.94		-113.98	-34.02	+79.96	-100.00	-399.96	-170.08	-136.06		-34.02	-113.98	-79.96	+50.00	-200.04
13	-399.96	-279.97		-119.99	-40.01	+79.98	-100.00	-419.98	-200.04	-160.03		-40.01	-119.99	-79.98	+50.00	-230.02
14	-419.98	-293.99		-125.99	-46.00	+79.99	-100.00	-439.99	-230.02	-184.02		-46.00	-125.99	-79.99	+50.00	-260.01
15	-439.99	-307.99		-132.00	-52.00	+80.00	-100.00	-459.99	-260.01	-208.01		-52.00	-132.00	-80.00	+50.00	-290.01
16	-459.99	-321.99		-138.00	-58.00	+80.00	-100.00	-479.99	-290.01	-232.01		-58.00	-138.00	-80.00	+50.00	-320.01
17	-479.99	-335.99		-144.00	-64.00	+80.00	-100.00	-499.99	-320.01	-256.01		-64.00	-144.00	-80.00	+50.00	-350.01
18	-499.99								-350.01							

TABLE IX-b.—Investments reduced in A, increased in B by smaller amount. Induced imports and exports. No saving.

is exactly the reciprocal of their marginal propensities to import (.3 : .2). The only stabilized item (besides the investment rates which were assumed as stable from the beginning) is the trade balance. From the 15th period on, it shows an export surplus in *A*, and an import surplus in *B*, of $80 per period.

What Model IX has taught us is that in the absence of induced saving a stable equilibrium of incomes can be reached

only if the increase in investment in one country is equal to the decrease in investment in the other country. In this case the trade balance will be perfectly adjusted to the changed levels of investment.

MODEL X: INVESTMENTS REDUCED IN A, INCREASED IN B; INDUCED SAVING, IMPORTS AND EXPORTS

We reintroduce now the assumption of positive marginal propensities to save. And we begin with the same set of propensities with which we began Model IV and Model VIII: $s_A = .2$, $m_A = .3$, $s_B = .1$, $m_B = .15$. Investments in A are assumed to be down by $100 per period, investments in B are up by the same amount. (Again, no changes in imports are directly associated with the changes in investment.)

Table X-a pictures what happens. Country A develops an export surplus, because its home-induced imports fall with falling incomes at home and its foreign-induced exports rise with rising incomes in the foreign country. Both the induced export surplus and the induced reductions in saving are off-sets to the autonomous reduction in investments. When the induced offsets become equal to the autonomous reduction in investments, the decline of incomes is halted, and incomes become stabilized at their reduced level. The opposite of all this takes place in B, only that, with the smaller propensities to save and to import, the induced offsets to the increased investment are less potent and can estop the rise in income only when income has reached a much higher level. The ultimate increment of income per period in B is twice as high as the ultimate decrement in A. This is clearly so because the marginal propensity to save in country B is half of what it is in A.

The sequence of income decrements in A shows a wave. Starting with the primary reduction of $100, income reaches

its low in the fourth period with a decrement of $147.63, and recovers afterwards slowly toward an ultimate decrement of $125. The reason for the wave is to be found in the fact that, from a certain point on, the foreign-induced increase in exports slightly overbalances the difference between the investment reduction and the home-induced seepages.[1]

Period	| COUNTRY A: s=0.2 M=0.3 Past ΔY (1)	Current ΔC (2)	Induced ΔS (3)	Home-Ind ΔM (4)	For-Ind ΔX (5) =12	Net Δ(X-M) (6) =-14	Auton. ΔI (7)	Current ΔY (8) =2+5+7	| COUNTRY B: s=0.1 M=0.15 Past ΔY (9)	Current ΔC (10)	Induced ΔS (11)	Home-Ind ΔM (12)	For-Ind ΔX (13) =4	Net Δ(X-M) (14) =-6	Auton. ΔI (15)	Current ΔY (16) =10+13+15
1							-100.00	-100.00							+100.00	+100.00
2	-100.00	-50.00	-20.00	-30.00	+15.00	+45.00	-100.00	-135.00	+100.00	+75.00	+10.00	+15.00	-30.00	-45.00	+100.00	+145.00
3	-135.00	-67.50	-27.00	-40.50	+21.75	+62.25	-100.00	-145.75	+145.00	+108.75	+14.50	+21.75	-40.50	-62.25	+100.00	+168.25
4	-145.75	-72.87	-29.15	-43.73	+25.24	+68.97	-100.00	-147.63	+168.25	+126.19	+16.82	+25.24	-43.73	-68.97	+100.00	+182.46
5	-147.63	-73.81	-29.53	-44.29	+27.37	+71.66	-100.00	-146.44	+182.46	+136.84	+18.25	+27.37	-44.29	-71.66	+100.00	+192.55
6	-146.44	-73.22	-29.29	-43.93	+28.88	+72.81	-100.00	-144.34	+192.55	+144.41	+19.26	+28.88	-43.93	-72.81	+100.00	+200.48
7	-144.34	-72.17	-28.87	-43.30	+30.07	+73.37	-100.00	-142.10	+200.48	+150.36	+20.05	+30.07	-43.30	-73.37	+100.00	+207.05
8	-142.10	-71.05	-28.42	-42.63	+31.06	+73.69	-100.00	-139.99	+207.05	+155.29	+20.71	+31.06	-42.63	-73.69	+100.00	+212.66
9	-139.99	-69.99	-28.00	-42.00	+31.90	+73.90	-100.00	-138.09	+212.66	+159.49	+21.27	+31.90	-42.00	-73.90	+100.00	+217.49
10	-138.09	-69.04	-27.62	-41.43	+32.62	+74.05	-100.00	-136.42	+217.49	+163.12	+21.75	+32.62	-41.43	-74.05	+100.00	+221.69
11	-136.42	-68.21	-27.28	-40.93	+33.25	+74.18	-100.00	-134.96	+221.69	+166.27	+22.17	+33.25	-40.93	-74.18	+100.00	+225.34
∞	-125.00	-62.50	-25.00	-37.50	+37.50	+75.00	-100.00	-125.00	+250.00	+187.50	+25.00	+37.50	-37.50	-75.00	+100.00	+250.00

TABLE X-a.—Investments reduced in A, increased in B by equal amount. Induced saving, imports and exports.

Owing to the induced saving in *B* and induced reduction in saving in *A*, the export surplus of *A* and import surplus of *B* do not become equal to the autonomous change in investment —and, thus, to the autonomous capital flow if it was a capital

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[1] We recall the rule we learned from Model IV that the income sequence will show a wave in that country which has the higher marginal propensity to save. (See p. 83 above.) But we must be careful not to apply this rule to the case of non-symmetrical changes in investment. There we shall find the more general rule that the wave occurs in the country in which the changes in income are absolutely smaller. For it is the repercussion of the larger changes in income abroad which, winning over the home-induced changes, is responsible for the wave.

flow which caused the equal and inverse changes in investment. To the autonomous shift of investment of $100 (from one country to the other) corresponds an ultimate trade balance of $75; in the fourth period, with $68.97, the trade balance comes into the 10-per cent-neighborhood of equilibrium.   But it must be borne in mind that this whole adjustment is the work of income effects alone.   Changes in relative prices, ruled out here as elsewhere, would possibly make the adjustment more perfect or perhaps reduce the concomitant changes in incomes.

We may contrast this case with one where, with the identical set of propensities, the changes in the current rates of investment are not symmetrical.   If no reduction at all occurs in the investments in A, while investments in B increase by $100 a period, incomes will rise in both countries.   For the sequences that result in this case we may go back to Table VIII-a (p. 118) which shows exactly what we wish to see.   At first thought it may appear peculiar that a table from Model VIII should fit a case under Model X.   Model VIII was concerned with foreign-induced exports from country A, brought about by an income expansion in B due to an autonomous increase in investment in B.   Model X is supposed to deal with inverse changes in investment in A and B such as might have been caused by autonomous capital movements from A to B.   Yet the difference between the two cases lies chiefly in the "antecedent actions" (in the "story before the curtain rises") which explain the why of the change in investment.   But the why of the change in investment is not significant for the sequences of incomes and trade balances which result from the changed investments.   Hence trade balances and incomes in A and B will show exactly the same developments whether, as it may be in Table VIII-a, the additional investment in B is undertaken as a part of an expansionary domestic policy or whether it is undertaken under the impact of an autonomous capital inflow.

The only material differences between these cases will be in the international balance of payments and in the reserve positions of the banking systems.   If an investment expansion in B takes place without any inflow of foreign capital, the banking system will have to meet the payments for the evolving import surplus out of old reserves or new foreign borrowing. If, on the other hand, a capital inflow into B has preceded the increased investment, the banking system will be well prepared to meet the payments for the ensuing import surplus. These matters, however, remain outside of the picture shown by our tables.   Therefore, the tables from Model VIII can fully illustrate those cases of Model X in which the reduction of investment in country A is zero.

Inasmuch as a detailed description of Table VIII-a has been given (p. 117) we may confine ourselves here to the remark that, if autonomous capital movements were at the root of the investment change, only a part-way adjustment of the trade balance to the capital flow could be claimed.   The trade balance becomes stabilized with an export surplus for A of $37.50 per period.[1]   On the other hand, advocates of capital exports who are anxious to demonstrate the benefits derived for both the capital-exporting and the capital-importing country, could hardly make up a more inviting and glowing picture.   Due to the lucky circumstances (1) that

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[1] If the *autonomous* foreign lending continues at the rate of $100 per period and the *net* foreign lending, with the stabilized export surplus, amounts to only $37.50 per period, the banking system of A must either engage in continual accommodating foreign borrowing—which would mean, practically, that the banking system of B is willing to increase steadily its holdings of balances in A-banks—or A must lose gold at a rate of $62.50 per period. This would have no primary deflationary effects (for, the undiminished level of home investment implied that those who made the foreign loans used inactive funds or new bank funds) but the reserve position of the banks in A, failing an offsetting policy of the monetary authorities, would deteriorate.   Hence the capital exports can continue in the long run only if country B permits an investment expansion in excess of the $100 borrowed from A.   (See below p. 173.)

investments were not encroached upon in A, (2) that investments were fully expanded in B, and (3) that the marginal propensity to save was fairly low in B, incomes were increased in both countries and by fairly substantial amounts. And, it might be added, if the capital-importing country is relatively poor and in need of capital, while capital and money markets of the capital-exporting country are in a liquid position, all these three circumstances may well be found to be extant in actual cases of the real world.

MORE VARIATIONS OF MODEL X

Changing the assumed set of propensities to $s_A = .2$, $m_A = .3$, $s_B = .2$, $m_B = .3$, we construct Table X-b for the case

	COUNTRY A : S = 0.2 M = 0.3								COUNTRY B : S = 0.2 M = 0.3							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
				Foreign Balance				Income from 2+5+7				Foreign Balance				Income from 10+13+15
	Spendable Income			=12	=-14				Spendable Income			=4	=-6			
Period	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Net Δ(X-M)	Auton. ΔI	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Net Δ(X-M)	Auton. ΔI	Current ΔY
1							-100.00	-100.00							+100.00	+100.00
2	-100.00	-50.00	-20.00	-30.00	+30.00	+60.00	-100.00	-120.00	+100.00	+50.00	+20.00	+30.00	-30.00	-60.00	+100.00	+120.00
3	-120.00	-60.00	-24.00	-36.00	+36.00	+72.00	-100.00	-124.00	+120.00	+60.00	+24.00	+36.00	-36.00	-72.00	+100.00	+124.00
4	-124.00	-62.00	-24.80	-37.20	+37.20	+74.40	-100.00	-124.80	+124.00	+62.00	+24.80	+37.20	-37.20	-74.40	+100.00	+124.80
∞	-125.00	-62.50	-25.00	-37.50	+37.50	+75.00	-100.00	-125.00	+125.00	+62.50	+25.00	+37.50	-37.50	-75.00	+100.00	+125.00

TABLE X-b.—Investments reduced in A, increased in B by equal amount. Induced saving, imports and exports.

of symmetrical changes in the rates of investment. We see that the approach toward equilibrium is smooth and fast. In comparison with case X-a we find that the ultimate values for country A are the same, whereas the income increase in B is now only half as large. No wonder, since the saving propensity is doubled.

With the same set of propensities as in X-b, Table X-c shows the sequences which result if investment in A is reduced by only \$50 and investment in B increased by \$100. The

interesting feature of this case is that the income in A at first shows negative changes, then recovers gradually, in order to reach, in the sixth period, approximately the level from which it started when the reduction of investments first occurred; and then it rises even above that level to approach an ultimate net increment of $31.25. It is the foreign-induced exports, due to country B's increasing incomes, which achieve this powerful

	COUNTRY A: S=0.2 M=0.3								COUNTRY B: S=0.2 M=0.3							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
				Foreign Balance				Income from 2+5+7				Foreign Balance				Income from 10+13+15
Period	Spendable Income			=12	=-14				Spendable Income			=4	=-6			
	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Net Δ(X-M)	Auton. ΔI	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Net Δ(X-M)	Auton. ΔI	Current ΔY
1							-50.00	-50.00							+100.00	+100.00
2	-50.00	-25.00	-10.00	-15.00	+30.00	+45.00	-50.00	-45.00	+100.00	+50.00	+20.00	+30.00	-15.00	-45.00	+100.00	+135.00
3	-45.00	-22.50	-9.00	-13.50	+40.50	+54.00	-50.00	-32.00	+135.00	+67.50	+27.00	+40.50	-13.50	-54.00	+100.00	+154.00
4	-32.00	-16.00	-6.40	-9.60	+46.20	+55.80	-50.00	-19.80	+154.00	+77.00	+30.80	+46.20	-9.60	-55.80	+100.00	+167.40
5	-19.80	-9.90	-3.96	-5.94	+50.22	+56.16	-50.00	-9.68	+167.40	+83.70	+33.48	+50.22	-5.94	-56.16	+100.00	+177.76
6	-9.68	-4.84	-1.94	-2.90	+53.33	+56.23	-50.00	-1.51	+177.76	+88.88	+35.55	+53.33	-2.90	-56.23	+100.00	+185.98
7	-1.51	-.76	-.30	-.45	+55.79	+56.24	-50.00	+5.03	+185.98	+92.99	+37.20	+55.79	-.45	-56.24	+100.00	+192.54
8	+5.03	+2.51	+1.01	+1.51	+57.76	+56.25	-50.00	+10.27	+192.54	+96.27	+38.51	+57.76	+1.51	-56.25	+100.00	+197.78
9	+10.27	+5.14	+2.05	+3.08	+59.33	+56.25	-50.00	+14.47	+197.78	+98.89	+39.56	+59.33	+3.08	-56.25	+100.00	+201.97
10	+14.47	+7.24	+2.89	+4.34	+60.59	+56.25	-50.00	+17.83	+201.97	+100.99	+40.39	+60.59	+4.34	-56.25	+100.00	+205.33
11	+17.83	+8.91	+3.57	+5.35	+61.60	+56.25	-50.00	+20.51	+205.33	+102.66	+41.07	+61.60	+5.35	-56.25	+100.00	+208.01
12	+20.51	+10.26	+4.10	+6.15	+62.40	+56.25	-50.00	+22.66	+208.01	+104.01	+41.60	+62.40	+6.15	-56.25	+100.00	+210.16
∞	+31.25	+15.62	+6.25	+9.38	+65.63	+56.25	-50.00	+31.25	+218.75	+109.37	+43.75	+65.63	+9.38	-56.25	+100.00	+218.75

TABLE X-c.—Investments reduced in A, increased in B by larger amount. Induced saving, imports and exports.

offset to the reduction of investments. Rather remarkable is how quickly the trade balances reach values near their equilibrium level—$54.00 in the third period, compared with an ultimate $56.25—whereas the changes in income are approaching their equilibrium very slowly. The income increments in B enter the neighborhood zone of equilibrium only in the 12th period; the income increments in A get there still more slowly (because of the small absolute values involved).

If the rate of domestic investment in A is not reduced at all, while in B additional investments of $100 per period are

undertaken, the income expanding effects will, of course, be greater in both countries, and the trade balance will be smaller. The sequences of this case can be found in Table VIII-b (p. 122).

Period	COUNTRY A: S=0.3 M=0.1 Spendable Income — Past ΔY (1)	Current ΔC (2)	Induced ΔS (3)	Foreign Balance =12 — Home-Ind ΔM (4)	For.-Ind ΔX (5)	=-14 Net Δ(X·M) (6)	Income from 2+5+7 — Auton. ΔI (7)	Current ΔY (8)	COUNTRY B: S=0.1 M=0.3 Spendable Income — Past ΔY (9)	Current ΔC (10)	Induced ΔS (11)	Foreign Balance =4 — Home-Ind ΔM (12)	For.-Ind ΔX (13)	=-6 Net Δ(X·M) (14)	Income from 10+13+15 — Auton. ΔI (15)	Current ΔY (16)
1							−100.00	−100.00							+100.00	+100.00
2	−100.00	−60.00	−30.00	−10.00	+30.00	+40.00	−100.00	−130.00	+100.00	+60.00	+10.00	+30.00	−10.00	−40.00	+100.00	+150.00
3	−130.00	−78.00	−39.00	−13.00	+45.00	+58.00	−100.00	−133.00	+150.00	+90.00	+15.00	+45.00	−13.00	−58.00	+100.00	+177.00
4	−133.00	−79.80	−39.90	−13.30	+53.10	+66.40	−100.00	−126.70	+177.00	+106.20	+17.70	+53.10	−13.30	−66.40	+100.00	+192.90
5	−126.70	−76.02	−38.01	−12.67	+57.87	+70.54	−100.00	−118.15	+192.90	+115.74	+19.29	+57.87	−12.67	−70.54	+100.00	+203.07
6	−118.15	−70.89	−35.44	−11.82	+60.92	+72.74	−100.00	−109.97	+203.07	+121.84	+20.31	+60.92	−11.82	−72.74	+100.00	+210.02
7	−109.97	−65.98	−32.99	−11.00	+63.01	+74.01	−100.00	−102.97	+210.02	+126.01	+21.00	+63.01	−11.00	−74.01	+100.00	+215.01
8	−102.97	−61.78	−30.89	−10.30	+64.50	+74.80	−100.00	−97.28	+215.01	+129.01	+21.50	+64.50	−10.30	−74.80	+100.00	+218.71
9	−97.28	−58.37	−29.18	−9.73	+65.61	+75.34	−100.00	−92.76	+218.71	+131.23	+21.87	+65.61	−9.73	−75.34	+100.00	+221.50
10	−92.76	−55.65	−27.83	−9.28	+66.45	+75.73	−100.00	−89.20	+221.50	+132.90	+22.15	+66.45	−9.28	−75.73	+100.00	+223.62
11	−89.20	−53.52	−26.76	−8.92	+67.09	+76.01	−100.00	−86.43	+223.62	+134.17	+22.36	+67.09	−8.92	−76.01	+100.00	+225.25
12	−86.43	−51.86	−25.93	−8.64	+67.57	+76.21	−100.00	−84.29	+225.25	+135.15	+22.53	+67.57	−8.64	−76.21	+100.00	+226.51
∞	−76.92	−46.15	−23.08	−7.69	+69.23	+76.92	−100.00	−76.92	+230.77	+138.46	+23.08	+69.23	−7.69	−76.92	+100.00	+230.77

TABLE X-d.—Investments reduced in A, increased in B by equal amount. Induced saving, imports and exports.

Comparing the end effects of X-b, X-c and VIII-b—the three cases which are based on the same set of propensities in both countries—we see the following magnitudes:

ΔI_A	ΔI_B	$\Delta(X-M)_{A,\infty}$	$\Delta Y_{A,\infty}$	$\Delta Y_{B,\infty}$	$\Delta Y_{A,\infty} + \Delta Y_{B,\infty}$
−100	+100	+75.00	−125.00	+125.00	—
− 50	+100	+56.25	+ 31.25	+218.75	+250.00
—	+100	+37.50	+187.50	+312.50	+500.00

We are hardly surprised to find that the algebraic sum of the income changes is exactly 5 times the algebraic sum of the

investment changes. For, with the trade balances cancelled out as the two countries are taken together, and with the marginal propensities to save uniformly .2, a joint multiplier of 5 was to be expected.

One other combination of propensities shall be tried: $s_A = .3$, $m_A = .1$, $s_B = .1$, $m_B = .3$. This combination was selected in order to have A, the country which exports capital

Period	COUNTRY A: S=0.3 M=0.1								COUNTRY B: S=0.1 M=0.3							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
					Foreign Balance			Income from 2+5+7					Foreign Balance			Income from 10+13+15
	Spendable Income				=12	=-14			Spendable Income				=4	=-6		
	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Net Δ(X-M)	Auton. ΔI	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Net Δ(X-M)	Auton. ΔI	Current ΔY
1							-50.00	-50.00							+100.00	+100.00
2	-50.00	-30.00	-15.00	-5.00	+30.00	+35.00	-50.00	-50.00	+100.00	+60.00	+10.00	+30.00	-5.00	-35.00	+100.00	+155.00
3	-50.00	-30.00	-15.00	-5.00	+46.50	+51.50	-50.00	-33.50	+155.00	+93.00	+15.50	+46.50	-5.00	-51.50	+100.00	+188.00
4	-33.50	-20.10	-10.05	-3.35	+56.40	+59.75	-50.00	-13.70	+188.00	+112.80	+18.80	+56.40	-3.35	-59.75	+100.00	+209.45
5	-13.70	-8.22	-4.11	-1.37	+62.83	+64.20	-50.00	+4.61	+209.45	+125.67	+20.95	+62.83	-1.37	-64.20	+100.00	+224.30
6	+4.61	+2.77	+1.38	+.46	+67.29	+66.83	-50.00	+20.06	+224.30	+134.58	+22.43	+67.29	+.46	-66.83	+100.00	+235.04
7	+20.06	+12.03	+6.02	+2.01	+70.51	+68.50	-50.00	+32.54	+235.04	+141.03	+23.50	+70.51	+2.01	-68.50	+100.00	+243.04
8	+32.54	+19.53	+9.76	+3.25	+72.91	+69.66	-50.00	+42.44	+243.04	+145.82	+24.31	+72.91	+3.25	-69.66	+100.00	+249.07
9	+42.44	+25.46	+12.73	+4.25	+74.72	+70.47	-50.00	+50.18	+249.07	+149.44	+24.91	+74.72	+4.25	-70.47	+100.00	+253.69
10	+50.18	+30.11	+15.05	+5.02	+76.11	+71.09	-50.00	+56.22	+253.69	+152.21	+25.37	+76.11	+5.02	-71.09	+100.00	+257.23
11	+56.22	+33.73	+16.87	+5.62	+77.17	+71.55	-50.00	+60.90	+257.23	+154.34	+25.72	+77.17	+5.62	-71.55	+100.00	+259.96
12	+60.90	+36.54	+18.27	+6.09	+77.99	+71.90	-50.00	+64.53	+259.96	+155.97	+26.00	+77.99	+6.09	-71.90	+100.00	+262.06
13	+64.53	+38.72	+19.36	+6.45	+78.62	+72.17	-50.00	+67.34	+262.06	+157.23	+26.21	+78.62	+6.45	-72.17	+100.00	+263.68
14	+67.34	+40.41	+20.20	+6.73	+79.10	+72.37	-50.00	+69.51	+263.68	+158.21	+26.37	+79.10	+6.73	-72.37	+100.00	+264.94
∞	+76.92	+46.15	+23.08	+7.69	+80.77	+73.08	-50.00	+76.92	+269.23	+161.54	+26.92	+80.77	+7.69	-73.08	+100.00	+269.23

TABLE X-e.—Investments reduced in A, increased in B by larger amount. Induced saving, imports and exports.

and suffers reductions in domestic investment, look more mature and more self-sufficient than B. Thus, the marginal propensity to save was assumed to be high in A and low in B, and the marginal propensity to import low in A and high in B. Three cases shall be tried with these propensities. In X-d investments are down $100 in A and up $100 in B. In X-e investments are down $50 in A and up $100 in B. And in X-f investments are down $100 in A and up $50 in B.

The sequences contain some peculiarities which would invite extended comments. However, we shall confine our attention to a few things: to the remarkable wave exhibited by the income series of A in Table X-d (starting with -100, going to -133 and settling down at -76.92); to the fact that the ultimate export surplus and the ultimate income decrement in A in Table X-d are both $76.92; to the fact that the ultimate

Period	COUNTRY A: S=0.3 M=0.1								COUNTRY B: S=0.1 M=0.3							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
				Foreign Balance				Income from 2+5+7				Foreign Balance				Income from 10+13+15
	Spendable Income			=12	=-14				Spendable Income			=4	=-6			
	Past ΔY	Current ΔC	Induced ΔS	Home Ind ΔM	For-Ind ΔX	Net Δ(X-M)	Auton. ΔI	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home Ind ΔM	For-Ind ΔX	Net Δ(X-M)	Auton. ΔI	Current ΔY
1							-100.00	-100.00							+50.00	+50.00
2	-100.00	-60.00	-30.00	-10.00	+15.00	+25.00	-100.00	-145.00	+50.00	+30.00	+5.00	+15.00	-10.00	-25.00	+50.00	+70.00
3	-145.00	-87.00	-43.50	-14.50	+21.00	+35.50	-100.00	-166.00	+70.00	+42.00	+7.00	+21.00	-14.50	-35.50	+50.00	+77.50
4	-166.00	-99.60	-49.80	-16.60	+23.25	+39.85	-100.00	-176.35	+77.50	+46.50	+7.75	+23.25	-16.60	-39.85	+50.00	+79.90
5	-176.35	-105.81	-52.90	-17.64	+23.97	+41.61	-100.00	-181.84	+79.90	+47.94	+7.99	+23.97	-17.64	-41.61	+50.00	+80.30
6	-181.84	-109.11	-54.55	-18.18	+24.09	+42.27	-100.00	-185.02	+80.30	+48.18	+8.03	+24.09	-18.18	-42.27	+50.00	+80.00
7	-185.02	-111.01	-55.51	-18.50	+24.00	+42.50	-100.00	-187.01	+80.00	+48.00	+8.00	+24.00	-18.50	-42.50	+50.00	+79.50
8	-187.01	-112.21	-56.10	-18.70	+23.85	+42.55	-100.00	-188.36	+79.50	+47.70	+7.95	+23.85	-18.70	-42.55	+50.00	+79.00
9	-188.36	-113.02	-56.51	-18.83	+23.70	+42.53	-100.00	-189.32	+79.00	+47.40	+7.90	+23.70	-18.83	-42.53	+50.00	+78.57
10	-189.32	-113.59	-56.80	-18.93	+23.57	+42.50	-100.00	-190.02	+78.57	+47.14	+7.86	+23.57	-18.93	-42.50	+50.00	+78.21
∞	-192.31	-115.39	-57.69	-19.23	+23.08	+42.31	-100.00	-192.31	+76.92	+46.15	+7.69	+23.08	-19.23	-42.31	+50.00	+76.92

TABLE X-f.—Investments reduced in A, increased in B by smaller amount. Induced saving, imports and exports.

income change in A in Table X-e is likewise $76.92, but this time positive instead of negative; to the fact that the income increase in B in Table X-f tends, among all numbers, toward the same $76.92; to the fine recovery of the income series in A in Table X-e (from -50.00 to $+76.92$); to the small wave shown by the income series of B in Table X-f (starting with $+50$, going to $+80.30$, ending up at $+76.92$); and to the fact that in all cases the neighborhood zone of equilibrium is reached much more quickly by the trade balance than by the income changes. The sequences can be more conveniently visualized on charts. The time series of Table X-d are
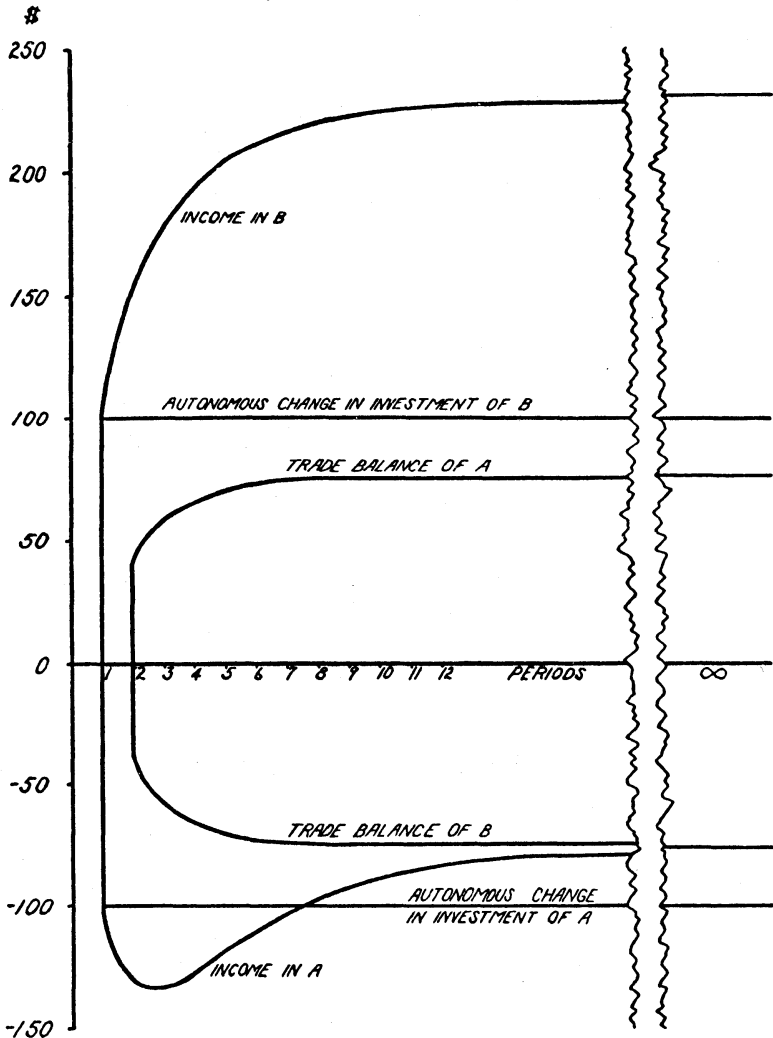
FIG. 6.—Time series of Table X-d.
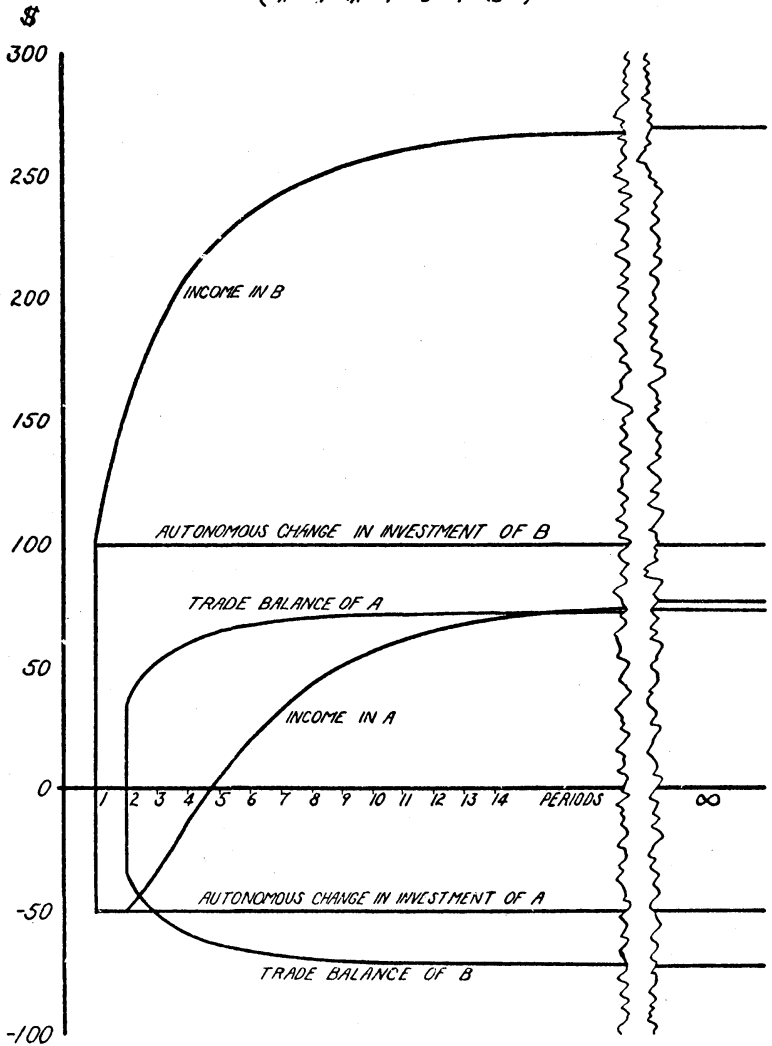
FIG. 7.—Time series of Table X-e.

reproduced graphically in Fig. 6, and those of Table X-e in Fig. 7.

Comparing the end-effects of the last three cases (arranged in the order of the combined net changes in investments), we see the following magnitudes:

ΔI_A	ΔI_B	$\Delta(X - M)_{A.\infty}$	$\Delta Y_{A.\infty}$	$\Delta Y_{B.\infty}$	$\Delta Y_{A.\infty} + \Delta Y_{B.\infty}$
−100	+ 50	+42.31	−192.31	+ 76.92	−115.39
−100	+100	+76.92	− 76.92	+230.77	+153.85
− 50	+100	+73.08	+ 76.92	+269.23	+346.15

No simple ratio between combined income changes and combined investment changes prevails here; this is clear, because the marginal propensities to save are different in the two countries. That the combined incomes rise (+153.85) when the combined investments are unchanged ($\Delta I_A = -100$, $\Delta I_B = +100$) is easily understood: investment is here shifted from a country with high to a country with low marginal propensity to save. The same reason explains why the combined income expansion with a net investment increase of $50 is so much greater than the combined income contraction with a net investment decrease of $50.

CHANGES IN BANK RESERVES

We must not lose sight of our many restricting assumptions. It is perhaps appropriate if we remind ourselves here of the fact that our analysis has failed throughout to take account of any effects which induced changes in the reserve position of banks may have on the course of events. This, of course, is quite in line with our assumption of "given interest rates" and with most of the reasoning in contemporary monetary theory. To what extent this defect invalidates or vitiates

multiplier analysis will be discussed later. Here we might try to get an idea of the quantitative significance which this neglected factor has in the sequences just studied.

Let us look into the effects which the happenings described in Table X-e would have upon the banking systems of A and B. Events will, of course, be different according as the sequences develop, on the one hand, out of an autonomous capital flow or, on the other hand, without such a flow. We shall first assume that an autonomous capital flow produces the changes in the rates of investment: people in A purchase $100 worth of newly floated B-securities per period. (The liquidity position in country A is such that domestic investment per period is reduced by $50, while investment outlays in B rise by the full $100). An export surplus develops in A which finally reaches $73.08 per period. "Equilibrium" is reached in both countries in the sense that—as long as the banking systems do all the accommodating lending and borrowing which is demanded—the money and income flows become stabilized. But what is implied in this accommodating lending and borrowing on the part of the banking systems?

The banking system in A, meeting the demand for foreign balances on the part of the capital exporters ($100) and absorbing the excess supply of foreign balances on the part of merchandise exporters (73.08), must engage in continual foreign borrowing in amounts of $26.92 per period. That is to say, either the holdings of foreign balances by the A-banking system (including the exchange stabilization fund) are permitted to decline, or the holdings of A-balances by the B-banking system (including the exchange stabilization fund) are built up. Or, instead of these movements of bank balances, gold may move from A to B—and must go on moving, period after period.

If the monetary authorities accept these movements of balances or gold with magnanimity, and offset their effects upon

the reserve positions of the commercial banks, things may continue as pictured.   Otherwise one of two things (or both) must happen.   Country B, with bank reserves steadily rising, may engage in an interest-induced expansion of investments (a "tertiary expansion" of the money supply[1]); this interest-induced increase in investments, which would be on top of the $100 increase assumed in Table X-e, would generate incomes in B in accordingly larger amounts, until the large gold reserves or "outside reserves" of B's banking system find use in payments for further increased imports.   Or, if the tertiary expansion in B could not be engineered, the point would eventually be reached when the monetary authorities in A could not stand further drains and the commercial banks in A could not stand further weakening of their reserve position. They would be forced into a restrictive credit policy.   And this might result in an interest-induced reduction of home investment (in excess of the $50 assumed in Table X-e) or in a reduction or discontinuation of the autonomous foreign lending.

If the changes in the investment levels in A and B were not the consequence of autonomous capital movements but took place without such capital flows, the effects upon the balance of payments would be different.   The drain on reserves would now occur in country B and the afflux in country A.   The induced foreign borrowing, or loss of gold, in country B would be $73.08 per period.   If country A could not quickly turn around and stage an expansion—and it would in many circumstances be doubtful that she could—country B, steadily losing reserves, would be forced—if exchange stability is to be maintained—to stop the expansion.

In all these cases we assumed that imports were changed only through changes in incomes; changes in investment did

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[1] See below p. 184, footnote 2.

not directly cause imports to change, according to our assumptions. (That is to say, investments consisted entirely in domestic outlays and did not require any imports for construction purposes.) If this assumption had not been made, the results would be different. If domestic investments required imported equipment or materials, the domestic income effects would be smaller and the increase in imports much larger. This would aggravate the deterioration of the balance of payments of country B in the case of an investment expansion which was not supported by autonomous capital inflow. And it would alleviate the deterioration of the balance of payments of country A in the case of an autonomous capital export.

There was a good reason for ruling out, in our models, such direct effects of investments upon imports. We would have had to make very arbitrary assumptions concerning the investment demand for imports. The portion of an additional investment which goes into imported equipment or materials depends entirely on the kind of investment which is undertaken. For ditch digging it is zero; for the equipment of existing plants with foreign machinery it is almost 100 per cent. Hence, while there may be enough sense in assuming "given" marginal propensities to import—after all, consumption habits do not change so quickly—, there is no sense in assuming a constant propensity to import for investment purposes.

That, in actual fact, there will usually be some import demand for investment purposes does not invalidate our analysis. While the sequences worked out in our models refer only to those disbursements which are made at home, one may supplement them by sequences worked out for that portion of the investment which is spent for imports. However, I have no intention to burden the reader with this additional task.

THE INVESTMENT MULTIPLIER INVOLVING TWO COUNTRIES

The equilibrium values in the sequences of Models IX and X were not arrived at by computing the whole geometric progression up to the magnitudes where further changes are confined to third or fourth decimal places. Instead, they were calculated with the help of new formulas. These formulas, as those before, were derived from equations expressing the equilibrium conditions in the two countries.

Equilibrium is reached when incomes no longer change. This will be the case when the autonomous investment in each country is completely offset by induced changes in saving, imports and exports. Thus (omitting again the Δ signs) we can write as the equilibrium condition for country A

$$(38) \qquad I_A = s_A Y_{A,\infty} + m_A Y_{A,\infty} - m_B Y_{B,\infty},$$

and for country B

$$(39) \qquad I_B = s_B Y_{B,\infty} + m_B Y_{B,\infty} - m_A Y_{A,\infty}.$$

From these two equations[1] we obtain in a few simple steps[2]

$$(41) \qquad Y_{A,\infty} = \frac{(s_B + m_B)I_A + m_B I_B}{(s_A + m_A)(s_B + m_B) - m_A m_B}.$$

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[1] Alternatively we can start with the two analogous equations, expressing that the income change must be equal to the sum of the primary investment, the induced consumption and the foreign-induced export. Thus, for country A,

$$Y_{A,\infty} = I_A + c_A Y_{A,\infty} + m_B Y_{B,\infty}.$$

Hence,

$$Y_{A,\infty}(1 - c_A) = I_A + m_B Y_{B,\infty}.$$

Since $1 - c_A = s_A + m_A$, this equation is identical with (38a) in the following footnote.

[2] $(38a) \qquad Y_{A,\infty}(s_A + m_A) = I_A + m_B Y_{B,\infty}$
$(39a) \qquad Y_{B,\infty}(s_B + m_B) = I_B + m_A Y_{A,\infty}$

$$(39b) \qquad Y_{B,\infty} = \frac{I_B + m_A Y_{A,\infty}}{s_B + m_B}.$$

Substituting (39b) in (38a), we get

Analogously,

$$(42) \qquad \Upsilon_{B,\infty} = \frac{m_A I_A + (s_A + m_A) I_B}{(s_A + m_A)(s_B + m_B) - m_A m_B}.$$

These formulas are designed for the general case where the changes in investment can have any different, positive or negative values. If the changes in investment in A and B are of equal magnitude and opposite signs, that is, if $I_B = -I_A$, the formulas will be a little simpler. Formula (41) becomes

$$(43) \quad \Upsilon_{A,\infty} = I_A \frac{s_B}{(s_A + m_A)(s_B + m_B) - m_A m_B}$$

$$= I_A \frac{1}{s_A + m_A + s_A \dfrac{m_B}{s_B}},$$

and (42) becomes

$$(44) \quad \Upsilon_{B,\infty} = -I_A \frac{s_A}{(s_A + m_A)(s_B + m_B) - m_A m_B}$$

$$= -I_A \frac{1}{s_B + m_B + s_B \dfrac{m_A}{s_A}}.$$

We are not surprised to find that these are exactly the formulas which apply to autonomous changes in foreign trade; that is to say, the multipliers in the case of equal and opposite changes in investment are the same as the autonomous foreign-trade multipliers.

It can be easily seen that these formulas would not work in calculations for cases under Model IX. In that model the

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$$(40) \qquad \Upsilon_{A,\infty}(s_A + m_A) = I_A + \frac{m_B I_B}{s_B + m_B} + \frac{m_A m_B \Upsilon_{A,\infty}}{s_B + m_B}$$

$$(40a) \qquad \Upsilon_{A,\infty}\left[\frac{(s_A + m_A)(s_B + m_B) - m_A m_B}{s_B + m_B}\right] = I_A + \frac{m_B I_B}{s_B + m_B}.$$

From this follows (41) in the text.

marginal propensities to save were zero. No determinate values can be obtained from the formulas if s_A and s_B are zero. And if, with the saving propensities zero, the investment change in A is not equal and opposite to that of B—as in case IX-b—the two equilibrium equations, (38) and (39), become inconsistent with each other. The economic reason for this was given above (p. 159): no equilibrium could be reached in that case.

But in case IX-a, when $I_B = -I_A$, and $s_A = s_B = 0$, equilibrium is reached. Yet our formulas refuse to work because, in this case, the two equations (38) and (39) become identical[1] and another equation is needed to solve for the two unknowns. This other equation is simply $Y_{A,\infty} = -Y_{B,\infty}$. For, obviously, since in the absence of induced saving the incomes are stabilized when the trade balances become equal to the investment changes, and the investment changes are equal and opposite by assumption, and the trade balances equal and opposite by definition, the income changes must also be equal and opposite. For this special case the formula becomes

$$(45) \qquad Y_{A,\infty} = I_A \frac{1}{m_A + m_B}.$$

In special cases, with equal and opposite investment changes in the two countries or with an investment change in only one country, the formulas are still such that one may reasonably call them "multipliers." Equations (43), (44) and (45) contain fractions which have the nature of multipliers. But the formulas (41) and (42) operate at the same time on two determinants, I_A and I_B. If one insisted on isolating "multi-

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[1] (38) $\qquad I_A = s_A Y_{A,\infty} + m_A Y_{A,\infty} - m_B Y_{B,\infty}$

(39) $\qquad I_B = s_B Y_{B,\infty} + m_B Y_{B,\infty} - m_A Y_{A,\infty}$

If the s_A and s_B are zero and $I_B = -I_A$, we have only one equation, namely,

$$I_A = m_A Y_{A,\infty} - m_B Y_{B,\infty}$$

pliers" he would have to have two multipliers for each country, one for the investment change at home, the other for the investment change abroad, and the resulting domestic income would be the sum of the two multiples.[1]

THE TRANSFER PROBLEM

Among the by-products of our analysis are a few clues which we have won for the so-called transfer problem.[2] The transfer problem, which occupied a conspicuous place in economic debates of the late twenties and early thirties on account of the German reparation payments, is, in briefest form and bared of several subtleties which were "infiltrated" later, the following: If a country is capable of raising, out of the given domestic income flow, the funds needed for the payments to foreign nations, will the transfer of these funds be trammeled or precluded sooner or later by a lack of foreign exchange?

The problem, unfortunately, was not quite so clearly formulated in the beginning. The presupposition, that the funds are raised in the paying country so as to reduce disbursements for consumption and home investment, was not always explicitly stated.[3] Undoubtedly, if no reduction of

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[1] Formula (41) would be transformed into

$$Y_{A,\bullet} = I_A \frac{s_B + m_B}{(s_A + m_A)(s_B + m_B) - m_A m_B} + I_B \frac{m_B}{(s_A + m_A)(s_B + m_B) - m_A m_B}.$$

[2] A part of what follows was anticipated by Mr. Lloyd A. Metzler in his article "The transfer problem reconsidered," *The Journal of Political Economy*, Vol. 50, 1942, pp. 397–414.

[3] Worse still, one of the provisions of the Dawes plan authorized the Reparations Agent to use, under certain conditions, collected funds for loans to German borrowers. Thus, funds would have actually been returned to the income stream after they had first been siphoned out. I emphasized in 1928 that such a policy was one among similar measures which could "sabotage" the transfer. *Cf.* my article "Währung und Auslandsverschuldung," *Mitteilungen des Verbandes österreichischer Banken und Bankiers*, Vol. 10, 1928, pp. 194 ff.

primary disbursements and incomes were brought about in the paying country, and no increase in disbursements in the receiving countries either, then the transfer problem would be hopeless from the beginning. If the fulfillment of the obligations against foreign nations is taken seriously at all, then the successful reduction of domestic purchasing power must be an unshakeable postulate.

On the other hand, the expansion of disbursements and incomes in the receiving countries cannot be relied upon. Hence, if one wishes to analyze the problem first on the basis of the least favorable assumptions, he will have to assume that disbursements in the receiving countries remain unchanged, while disbursements in the paying country are reduced by the full amount of the fixed obligations.

The next question is then how much of an adjustment of the trade balance can be expected under the assumed circumstances to result from income effects alone. The paying country probably has some stock of gold or foreign exchange to start with. But for a continuous transfer, to be maintained period after period, year after year, only an export surplus can provide the necessary foreign exchange.[1] In general, the export surplus can be created through income effects, price effects, and exchange rate effects.[2] Exchange rate effects are definitely ruled out as incompatible with the

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[1] An inflow of loans and investments from abroad may take the place of an export surplus. In the case of Germany this actually happened. The Germans received more foreign loans than they paid as reparations. Since they have not repaid the loans, they have made a net profit from these transactions.

[2] Besides these three, there may be a primary investment effect. Reduced investments in the paying country may involve a reduced import demand for investment (construction) purposes; increased investments in the receiving country may involve an increased import demand for investment (construction) purposes. The former results in reduced imports, the latter in increased exports of the paying country. Even if we chose to rule out these primary investment effects upon foreign trade in our analysis, they still operate in reality.

postulate of stable currencies. Price effects are provisionally ruled out, i.e., stable prices are assumed, so that we can isolate the income effects and are able to see what portion of the "levy" can be produced in foreign funds without any change in the barter terms of trade.

THE TRANSFERABLE PORTION OF THE LEVY WITHOUT PRICE CHANGES

Let us call the reduction in disbursements for consumption and home investment in the paying country, which must be equal to the imposed obligations to foreign nations, briefly "the levy." Now, given the levy, which is denoted by $-I_A$, given also s_A, m_A, s_B and m_B, and assumed that no changes occur in primary disbursements abroad, i.e., $I_B = 0$, what will be the ratio between the eventually resulting trade balance, $m_B Y_{B,\infty} - m_A Y_{A,\infty}$, and the levy, $-I_A$?

Using our formulas of the home-investment multipliers involving two countries, (41) and (42), but taking into account that $I_B = 0$, we may write

$$(46) \qquad Y_{A,\infty} = I_A \frac{s_B + m_B}{(s_A + m_A)(s_B + m_B) - m_A m_B},$$

and

$$(47) \qquad Y_{B,\infty} = I_A \frac{m_A}{(s_A + m_A)(s_B + m_B) - m_A m_B}.$$

Hence, the eventual export surplus of A,

$$(48) \quad m_B Y_{B,\infty} - m_A Y_{A,\infty} = I_A \frac{-s_B m_A}{(s_A + m_A)(s_B + m_B) - m_A m_B}.$$

For the portion which the export surplus is of the levy we obtain

$$(49) \qquad \frac{m_B Y_{B,\infty} - m_A Y_{A,\infty}}{-I_A} = \frac{s_B m_A}{s_A s_B + s_A m_B + s_B m_A}.$$

In this fraction the denominator exceeds the numerator by $s_A(s_B + m_B)$. If m_B is the marginal propensity of the rest of

the world to import from country A, then it will surely be negligible. Thus, it is $s_A s_B$ which reduces the transfer possibilities created through income effects.

If s_A, the marginal propensity to save in the paying country is zero, the denominator becomes equal to the numerator, and an export surplus will develop which is equal to the levy. In plain words this means: if the decline in income does not lead to an induced reduction in savings in the paying country, then income will continue to decline until a sufficient export surplus is created (in spite of somewhat declining exports)[1] simply through induced reduction of imports.

Nobody will contend that this is a very hopeful or desirable state of affairs. Yet, one must insist that in this case the transfer problem is "solved" completely, in that one hundred per cent of the "levy" becomes available in foreign exchange derived from an induced export surplus. In actual fact, however, this solution of the transfer problem would possibly be identical with a breakdown of the budgetary problem of "internal collection." That is to say, it would become impossible to raise by means of taxation the necessary funds to meet the obligations: domestic funds would be lacking, not foreign exchange. The national income—money income being a perfect measure of real income since all prices are by assumption unchanged—might be so much reduced that the required funds could no longer be extracted from the impoverished population. Those who denied the existence of an independent transfer problem would, in this case, be vindicated: the transfer problem would be "automatically" solved to the same extent to which the budgetary problem could be solved.

So much about the case where s_A is zero. If the marginal propensity to save is not zero in the paying country, so that

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[1] If m_B is zero, exports remain at their level. But if m_B is anything above zero, the reduction in foreign income, which follows from the diminished imports of the paying country, must somewhat reduce foreign purchases from the paying country.

induced reductions in saving arrest the decline in income, then only less than one hundred per cent of the levy can be produced in foreign means of payments with unchanged terms of trade. The ratio of the induced export surplus to the levy will depend (assuming that m_B is negligible) on the magnitude of $s_A s_B$ in relation to the magnitude of $s_B m_A$, or, that is, on the relative sizes of s_A and m_A. The larger m_A in relation to s_A, the greater the portion of the levy which becomes available in foreign funds. If s_A and m_A are equal, one half the levy becomes transferable without resort to gold stocks and without price changes.

To express it somewhat more exactly, we may state that, if the marginal propensity of the rest of the world to import from country A is negligible, i.e. if $m_B = 0$, formula (49) becomes

$$(50) \qquad \frac{m_B \Upsilon_{B,\infty} - m_A \Upsilon_{A,\infty}}{-I_A} = \frac{s_B m_A}{s_A s_B + s_B m_A},$$

which is equal to $\dfrac{m_A}{s_A + m_A}$ or $\dfrac{1}{\dfrac{s_A}{m_A} + 1}$.

The common sense of it is this: The export surplus without price changes (and without foreign income expansion) can be created only through a fall in imports. Hence, the greater the marginal propensity to import, the greater will be the transferable portion of the levy. The greater the marginal propensity to save, the smaller will be the transferable portion of the levy, but the smaller will be also the secondary decline in income which results from the levy, i.e., from the reduction in primary disbursements.

ALTERNATIVE ROUTES FROM HERE ON

Although this is about all that multiplier analysis has to contribute to the setting of the transfer problem of unilateral payments, it would be hardly excusable to leave the subject

at this point, without any send-off or suggestions as to where to go from here.

It is a matter of taste whether we drop first the assumption that prices remain unchanged or whether we choose as our next step to allow foreign disbursements to increase.   The one alternative would add price effects—changes in the terms of trade—to the income effects operating in the paying country, but would not yet allow changes in demand in the receiving country.   The other alternative would add income effects in the receiving country to those in the paying country, but would not yet allow changes in relative prices and their effects on trade.[1]

If the second alternative is preferred, we need merely refer to some cases of our Model X, showing the effects of inverse changes in investments in two countries.   Tables X-a, X-b, X-d, and X-f would all be possible variations of the transfer case, with fully effective levy in the paying country and either full or only part-way expansion of disbursements in the receiving country.   The transferable portion of the levy was 75 per cent and over, in the first three of the four cases, but only some 42 per cent in the case described in Table X-f.   In no case did the export surplus which could be achieved merely through income effects reach the full amount of the levy.[2]   Only in

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[1] Readers versed in the literature of the field will recognize in the choice between the alternatives the main issues of the transfer controversy of the late twenties and early thirties.   It was Lord Keynes who concentrated on the price effects and refused to consider income effects in the receiving country. (J. M. Keynes, "The German transfer problem" *Economic Journal*, Vol. 39, 1929, p. 1; "A rejoinder," *ibid.* p. 179; "A reply," *ibid.* p. 404.)   It was Professor Ohlin who concentrated on the income effects in both countries and minimized the price effects.   (Bertil Ohlin, "Transfer difficulties, real and imagined," *Economic Journal*, Vol. 39, 1929, p. 172; "Mr. Keynes' views on the transfer problem," *ibid.* p. 400.)

[2] If there are primary investment effects, e.g. if the receiving country buys from the paying country equipment or materials needed for increased investment, the final export surplus will be greater, although the income effects will of course be smaller.

case IX-a was a one-hundred-per cent transfer possible, because of zero marginal propensities to save.[1]

If one follows the first alternative rather than the second, and looks into price effects while leaving primary disbursements in the receiving country unchanged, he is more in line with our original intention to analyze the problem on the basis of the least favorable assumptions.

Changes in prices are fairly certain to occur when incomes change. About the likelihood and the extent of such price changes more will be said later. At this point we simply take it for granted that in our case of an income reduction—both primary and secondary (and also tertiary if the banking system restricts credit when its reserves dwindle[2])—prices are liable to fall. The effects of this price fall may be felt in exports or imports or in both. For some reason, the effects upon exports are usually, in discussions of the problem, given much more attention than the effects upon imports.

The price effects upon exports are discussed in terms of the elasticity of foreign demand for the products and services of the paying country. Should this elasticity be less than unity —with foreign incomes assumed not to be expanded—the total value of exports would fall rather than rise in consequence of the price reductions. This "possibility" was the quintes-

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[1] Lloyd A. Metzler, in the article mentioned, demonstrates cases where full transfer is achieved merely through income effects in spite of positive saving propensities. But Dr. Metzler reaches his conclusions only by assuming a marginal propensity to invest which raises the marginal propensity to spend to a figure equal to or in excess of 1. In other words, the marginal propensity to invest which Dr. Metzler assumes in the particular cases offsets or overcompensates the marginal propensity to save.

[2] It has become customary to call the reserve-induced expansion or contraction of bank credit the "secondary" expansion or contraction. (Viner *op. cit.* p. 395, and many others.) However, since we have used the term secondary for those income changes which arise in connection with the re-spending (failure to spend) consequential upon the increase (decrease) of primary disbursements, we must demote the bank-induced changes in disbursements and income to the rank of tertiary changes.

sence of the pessimistic transfer theories of Lord Keynes and
his followers. The degree of probability that the foreign
demand for the products of the paying country should, in
actual fact, fail to be elastic was not examined with sufficient
care. This probability, as I have pointed out elsewhere,[1]
is not great. Firstly, one must consider that of the various
exports of the paying country not all are likely to be a large
share of the world supply of the goods in question; thus, even
if the total demand for certain goods is inelastic, the demand
for the particular product of one country in competition with
the same (similar) products of all other countries is apt to be
elastic, perhaps even *very* elastic. Secondly, one must bear
in mind that not only actual export articles but also potential
export articles count when the possibilities of an increase of
exports are scanned; the elasticity of foreign demand for a
good of which nothing has been exported hitherto, may be
"infinity"—that is to say, a small price reduction may make
the good exportable.

So much concerning price-induced exports. What about
the probability of a price-induced fall in imports? The
prices of import articles are not changed, for it was assumed
that no expansion of foreign incomes occurs. The reduction
of imports through the domestic fall of incomes has already
been fully accounted for. But there is yet another force
reducing the domestic demand for imports: price reductions
of domestic goods. We know that large groups of goods
(groups—rather than particular goods) are always competing,
not complementary, with one another.[2] Thus, that the mass
of domestic goods and services is competing with the mass of
imported ones is undoubted. Therefore, if prices of domestic

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[1] "The theory of foreign exchanges," *Economica*, 1939, pp. 382–83;
"Transfer und Preisbewegung," *Zeitschrift für Nationalökonomie*, Vol. I, 1930,
p. 557.
[2] J. R. Hicks, *Value and Capital*, Oxford, 1939, p. 50.

goods and services fall, substitutions of domestic for foreign goods will take place. At the changed relative prices, the total (and average) propensity to import cannot remain at its former level, that is to say, we must expect a price-induced fall in imports on top of the income-induced fall in imports.

Of course, incomes will not fall as much as they would in the absence of price changes. For, the sequence of income reductions and the eventual income level depend, among other things, upon the development of foreign trade. If price-induced improvements of the trade balance appear in the picture, the fall in incomes will be stopped at an earlier point. This means also that the income-induced fall in imports, if accompanied by a price-induced fall in imports, will not be as heavy as it would be without price changes. But the two together will certainly be much heavier than one alone. Hence, with the fall in domestic prices, the arising export surplus will be greater, and the income decline smaller, than was shown before in Model X. If all three factors—income-induced fall in imports, price-induced fall in imports, and price-induced increase in exports—are pulling together, it is not impossible that one-hundred per cent of the levy can be transferred without too much of a secondary income contraction in the paying country.

Should now also the conditions abroad be less adverse than was assumed, in that expansions of primary disbursements in the receiving countries take place, then there is still less ground for a "pessimistic" transfer theory. With income expansions abroad, a foreign-induced increase in exports will join the other three factors which have just been named as the forces combining in the creation of an export surplus for the paying country. Should price effects in the income-expanding, receiving countries also join in, so that the price-induced improvements of the trade balance of the paying country can be that much greater, any remaining reason for transfer

pessimism will be displaced by very comforting thoughts. It may become quite possible that the ultimate income reduction in the paying country is less than the levy—see, for instance, Table X-d—and at the same time, the export surplus, with the aid of price effects, equal to, or above, the levy. And if the price changes are of a peculiar type,[1] the barter terms of trade may be improved rather than deteriorated.

This is not the place to expatiate on these points. We leave the subject with the conclusions that there is no *a priori* reason for believing the transfer problem to be a very grave or even insoluble one; that an expansion of primary disbursements in the receiving countries can do much to remove or ease all transfer difficulties; and that flexibility of prices, and particularly of costs, can greatly alleviate the squeeze in the paying country.

PARALLEL INVESTMENT EXPANSIONS WITH BALANCED TRADE

Another by-product of our analysis helps clarify an old question, to which a quantitative answer, to my knowledge, has never been attempted. I refer to the problem of synchronized adjustment of a monetary expansion in one country to simultaneous expansions which are being undertaken abroad, an adjustment in such a way that the "balance of payments" is not disturbed.

The problem was usually dealt with in discussions of the gold standard mechanism. There we were told that a nation indulging in a credit expansion, while other nations were abstaining in this respect, would be liable to lose gold. The next step was to explain that in a case of parallel expansion in

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[1] The export goods of the paying country may rise in price in consequence of the foreign-induced increase in exports, while the import goods of the paying country may fall in price in consequence of the home-induced fall in imports. Cf. Gottfried von Haberler, *The Theory of International Trade*, pp. 69 and 75–76.

all nations none of them would have to lose gold, provided that the expansions were moving in pace with one another. Nations which proceeded faster than the others in the creation of credit would suffer an outflow of gold; nations which fell behind the others would attract gold from abroad. Qualifications were usually made to the effect that "equal pace," of course, should not mean *equal* amounts of additional credit but should mean *corresponding* amounts. Yet, just what these "corresponding" amounts were, or how the "correct" magnitudes or proportions were determined, remained obscure.

The analysis of this chapter throws some light on the matter. (That we now speak of increased primary disbursements or investment, where we once spoke of credit expansion, or that we now speak of gold, foreign balances and induced foreign borrowing, where we once spoke merely of gold, makes little difference.) We found that the ratio of the induced change in the trade balance to the autonomous changes in home investment depended on the marginal propensities to save and to import in the countries concerned. Hence, for every given set of marginal propensities there must be between the increments in the countries' home investment a certain proportion for which the induced change in the trade balance is zero. Speaking in terms of two countries only, the investment increase in country A in relation to a given investment increase in country B may be too large, so that an import surplus arises in country A; or the investment increase in A may be too small, so that an export surplus arises; *one* proportion between the investment increase in A and the investment increase in B will exist, for which induced exports and imports balance each other—and this proportion is determined by the relative marginal propensities to save and to import in A and in B. Let us repeat, lest we forget it, that income effects alone are again supposed to be operating, price changes and price effects being ruled out.

Very simple literary and algebraic reasoning leads us to the exact expression for the proportion, q, of A's investment increase to B's investment increase for which the induced trade balance is zero. In algebraic symbols this setting of the problem reads:

(51) $$I_A = qI_B,$$

and

(52) $$m_A Y_{A,\infty} = m_B Y_{B,\infty}.$$

Now, if changes in exports and imports balance, the conditions for equilibrium are that induced savings must be equal to autonomous changes in investment in each country (for, otherwise, incomes would continue to change). That is to say,

(53) $$I_A = s_A Y_{A,\infty},$$

and

(54) $$I_B = s_B Y_{B,\infty}.$$

Since q is the ratio between I_A and I_B, we have

(55) $$q = \frac{s_A Y_{A,\infty}}{s_B Y_{B,\infty}}.$$

From (52) we know that

(52a) $$\frac{Y_{A,\infty}}{Y_{B,\infty}} = \frac{m_B}{m_A}.$$

Hence,

(56) $$q = \frac{s_A}{s_B} \frac{m_B}{m_A}.$$

This is the ratio between the parallel expansions without disturbance of the trade balance. The income increments which will correspond to the investment expansions in the

two countries can be computed from the formulas (41) and (42).

MODEL XI: INVESTMENTS INCREASED IN A AND B IN A PROPORTION WHICH EQUALIZES INDUCED IMPORTS

In Model X the effects were shown which *inverse* changes in home investment in A and B would have upon trade balances and incomes. It does not seem necessary to present another model to show the effects which *parallel* changes in home investment in A and B would have upon trade balances and incomes. The principles operating in these two cases are too similar. But we shall present a model for the special case which we have just discussed, the case where the parallel expansions of investments are so well tuned to each other that no lasting disturbance of the trade balance arises.

Model XI is therefore designed to show the sequences which are started by an increase of B's home investment by an amount of \$100, and a simultaneous increase in A's home investment by such an amount as is calculated to result in a perfect balancing of induced imports. The exact amount of A's investment increase, of course, depends on the marginal propensities to save and to import in A and in B, and is in each case computed with the aid of formula (56).

The propensities in country A are assumed to be again $s_A = .2$ and $m_A = .3$. The marginal propensities in B shall first be $s_B = .2$ and $m_B = .2$. According to formula (56), $q = \frac{2}{3}$; that is to say, the "correct" amount of home investment in A—the amount which "corresponds" to the 100 dollar increase in B—is \$66.67 per period. Table XI-a is constructed on this basis.

Both countries, of course, have rising incomes, rising savings, rising imports, and rising exports. The end of the growth in all these items will be reached when induced savings are up

to the level of the autonomous investment increments. The import increments, in Table XI-a, are equal in A and B, i.e., the trade balances are zero, not only at the ultimate equilibrium level but throughout the entire adjustment period. We understand why this must be so: with s_A equal to s_B, the additional investments in A and B were made inversely proportional to m_A and m_B. Hence, $m_A I_B$ is equal to $m_B I_A$ and the induced imports must be equal in A and B.

	COUNTRY A: S=0.2 M=0.3								COUNTRY B: S=0.2 M=0.2							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	K=5.00		Foreign Balance					Income from 2+5+7	K=5.00		Foreign Balance					Income from 10+13+15
Period	Spendable Income			=12	=-14				Spendable Income			=4	=-6			
	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For.-Ind ΔX	Net Δ(X-M)	Auton. ΔI	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For.-Ind ΔX	Net Δ(X-M)	Auton. ΔI	Current ΔY
1							66.67	66.67							100.00	100.00
2	66.67	33.34	13.33	20.00	20.00	0	66.67	120.00	100.00	60.00	20.00	20.00	20.00	0	100.00	180.00
3	120.00	60.00	24.00	36.00	36.00	0	66.67	162.67	180.00	108.00	36.00	36.00	36.00	0	100.00	244.00
4	162.67	81.34	32.53	48.80	48.80	0	66.67	196.80	244.00	146.40	48.80	48.80	48.80	0	100.00	295.20
5	196.80	98.40	39.36	59.04	59.04	0	66.67	224.11	295.20	177.12	59.04	59.04	59.04	0	100.00	336.16
6	224.11	112.06	44.82	67.23	67.23	0	66.67	245.96	336.16	201.70	67.23	67.23	67.23	0	100.00	368.93
7	245.96	122.98	49.19	73.79	73.79	0	66.67	263.44	368.93	221.36	73.78	73.79	73.79	0	100.00	395.15
8	263.44	131.72	52.69	79.03	79.03	0	66.67	277.42	395.15	237.09	79.03	79.03	79.03	0	100.00	418.12
9	277.42	138.71	55.48	83.23	83.23	0	66.67	288.61	416.12	249.67	83.22	83.23	83.23	0	100.00	432.90
10	288.61	144.31	57.72	86.58	86.58	0	66.67	297.56	432.90	259.74	86.58	86.58	86.58	0	100.00	446.32
11	297.56	148.78	59.51	89.27	89.27	0	66.67	304.72	446.32	267.79	89.26	89.27	89.27	0	100.00	457.06
∞	333.33	166.66	66.67	100.00	100.00	0	66.67	333.33	500.00	300.00	100.00	100.00	100.00	0	100.00	500.00

TABLE XI-a.—Investments increased in A and in B with induced imports exactly balancing.

The countries will neither lose nor gain gold in the process of monetary expansion, nor will any induced foreign lending or borrowing take place. While the volume of foreign trade expands, the balance is always zero. The ultimate income increment in A is \$333.33, which with the \$66.67 investment increment corresponds to an investment multiplier of 5. The multiplier in country B is likewise 5. The reason is fairly obvious. Since in all cases of Model XI the investments are "tuned" in a way that home-induced imports and foreign-

induced exports are equal in equilibrium, if not throughout, the effects of the import propensities and foreign repercussions upon incomes must cancel out. The ultimate income levels, therefore, are only a function of the marginal propensities to save. In case XI-a, with $s_A = .2$ and $s_B = .2$, the multipliers must be 5 in both countries.

It may be noted that the technique used by certain statistical multiplier computers[1] would give entirely different results, in that export increments would not be recognized as induced changes and, thus, would be parts of the multiplicands. Additional investments plus exports would be $166.67 in A and $200.00 in B, and the corresponding multipliers would be 2 in A and 2.5 in B.

In Table XI-b we let s_B be only .1, while m_B is .2 as before. The lower marginal propensity to save in B, which will permit her to reach higher income levels and, thus, higher imports, allows country A to engage in a greater expansion. Indeed, with s_B half of what it was before, I_A can be twice that of before; the investment increment in A ($133.33) is now larger than that of B ($100.00).

In this case country A is not entirely free from temporary drains on gold reserves or from induced foreign borrowing. For there will be small and diminishing import surpluses during the transition period. We remember that even temporary trade balances were avoided in case XI-a, because there the marginal propensities to save were equal in the two countries. Now country A is engaging in much larger investments, and the lower savings propensity of B, which justifies A's larger investments, is not giving quick results. To be sure, B's imports eventually increase twice as much as in the case before (to $200.00), but they start out with the same first dose as before ($20.00). Hence country A, with imports starting out at twice the size as before ($40.00), gets an import

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[1] See above, p. 47.

surplus of $20.00 in the second period, and even $24.00 in the third period; the import surpluses thereafter become smaller and smaller, and eventually disappear entirely.

The temporary disturbance in the balance of payments cannot be avoided in this case except by a very "scientific" dosing of home investments in *A*.   One might start with smaller doses

Period	COUNTRY A : S=0.2 M=0.3								COUNTRY B : S=0.1 M=0.2							
	K=5.00			Foreign Balance				Income from 2+5+7	K=10.00			Foreign Balance				Income from 10+13+15
	Spendable Income			=12	=-14				Spendable Income			=4	=-6			
	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For.Ind ΔX	Net Δ(X-M)	Auton. ΔI	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For.Ind ΔX	Net Δ(X-M)	Auton. ΔI	Current ΔY
1							133.33	133.33							100.00	100.00
2	133.33	66.67	26.66	40.00	20.00	-20.00	133.33	220.00	100.00	70.00	10.00	20.00	40.00	+20.00	100.00	210.00
3	220.00	110.00	44.00	66.00	42.00	-24.00	133.33	285.33	210.00	147.00	21.00	42.00	66.00	+24.00	100.00	313.00
4	285.33	142.66	57.07	85.60	62.60	-23.00	133.33	338.59	313.00	219.10	31.30	62.60	85.60	+23.00	100.00	404.70
5	338.59	169.29	67.72	101.58	80.94	-20.64	133.33	383.56	404.70	283.29	40.47	80.94	101.58	+20.64	100.00	484.87
6	383.56	191.78	76.71	115.07	96.97	-18.10	133.33	422.08	484.87	339.41	48.49	96.97	115.07	+18.10	100.00	554.48
7	422.08	211.04	84.42	126.62	110.90	-15.72	133.33	455.27	554.48	388.13	55.45	110.90	126.62	+15.72	100.00	614.75
8	455.27	227.64	91.05	136.58	122.95	-13.63	133.33	483.92	614.75	430.33	61.47	122.95	136.58	+13.63	100.00	666.91
9	483.92	241.96	96.78	145.18	133.38	-11.80	133.33	508.67	666.91	466.84	66.69	133.38	145.18	+11.80	100.00	712.02
10	508.67	254.34	101.73	152.60	142.40	-10.20	133.33	530.07	712.02	498.42	71.20	142.40	152.60	+10.20	100.00	751.02
11	530.07	265.04	106.01	159.02	150.20	-8.82	133.33	548.57	751.02	525.72	75.10	150.20	159.02	+8.82	100.00	784.74
12	548.57	274.29	109.71	164.57	156.95	-7.62	133.33	564.57	784.74	549.32	78.47	156.95	164.57	+7.62	100.00	813.89
13	564.57	282.29	112.91	169.37	162.78	-6.59	133.33	578.40	813.89	569.72	81.39	162.78	169.37	+6.59	100.00	839.09
14	578.40	289.20	115.68	173.52	167.82	-5.70	133.33	590.35	839.09	587.36	83.91	167.82	173.52	+5.70	100.00	860.88
15	590.35	295.17	118.07	177.11	172.18	-4.93	133.33	600.68	860.88	602.61	86.09	172.18	177.11	+4.93	100.00	879.72
16	600.68	300.34	120.14	180.20	175.94	-4.26	133.33	609.61	879.72	615.81	87.97	175.94	180.20	+4.26	100.00	896.01
∞	666.67	333.34	133.33	200.00	200.00	0	133.33	666.67	1,000.00	700.00	100.00	200.00	200.00	0	100.00	1,000.00

TABLE XI-b.—Investments increased in A and in B with induced imports exactly balancing.

of additional investment and then gradually increase them in accordance with the development of incomes and imports in *B*.   We shall not take here the time to construct a model for such a scheme of gradually stepped-up investments.   If, as is done in Table XI-b, the investment level in country *A* is chosen from the outset at that height which in equilibrium will give a perfect balance of induced exports and imports, one must reckon with the temporary disturbance of the bal-

ance during the adjustment period. But the total loss of gold reserves or the total amount of induced foreign borrowing is very modest in comparison with total investments and, especially, with the generated incomes. For the first two years—eight periods—the cumulated import surplus is $135.09, which compares with a cumulated investment of $1066.67 and a cumulated income increment of $2722.08.

	COUNTRY A :		S=0.2	M=0.3					COUNTRY B :		S=0.3	M=0.2			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	K=5.00		Foreign	Balance				Income from 2+5+7	K=3.33		Foreign	Balance				Income from 10+13+15
	Spendable. Income		=12	=-14					Spendable Income		=4	=-6			
Period	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Net Δ(X-M)	Auton. ΔI	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Net Δ(X-M)	Auton. ΔI	Current ΔY
1							44.44	44.44							100.00	100.00
2	44.44	22.22	8.89	13.33	20.00	+6.67	44.44	86.66	100.00	50.00	30.00	20.00	13.33	-6.67	100.00	163.33
3	86.66	43.33	17.33	26.00	32.67	+6.67	44.44	120.44	163.33	81.66	49.00	32.67	26.00	-6.67	100.00	207.66
4	120.44	60.22	24.09	36.13	41.53	+5.40	44.44	148.19	207.66	103.83	62.30	41.53	36.13	-5.40	100.00	239.96
5	146.19	73.09	29.24	43.86	47.99	+4.13	44.44	165.52	239.96	119.98	71.99	47.99	43.86	-4.13	100.00	263.84
6	165.52	82.76	33.10	49.66	52.77	+3.11	44.44	179.97	263.84	131.92	79.15	52.77	49.66	-3.11	100.00	281.58
7	179.97	89.99	35.99	53.99	56.32	+2.33	44.44	190.75	281.58	140.79	84.47	56.32	53.99	-2.33	100.00	294.78
8	190.75	95.37	38.15	57.23	58.96	+1.73	44.44	198.77	294.78	147.39	88.43	58.96	57.23	-1.73	100.00	304.62
9	198.77	99.39	39.75	59.63	60.92	+1.29	44.44	204.75	304.62	152.31	91.39	60.92	59.63	-1.29	100.00	311.94
10	204.75	102.38	40.95	61.42	62.39	+.97	44.44	209.21	311.94	155.97	93.58	62.39	61.42	-.97	100.00	317.39
∞	222.22	111.11	44.44	66.67	66.67	0	44.44	222.22	333.33	166.66	100.00	66.67	66.67	0	100.00	333.33

TABLE XI-c.—Investments increased in A and in B with induced imports exactly balancing.

The temporary disturbance of the balance of payments need not always be an "unfavorable" one. In Table XI-c we encounter a "favorable" disturbance. In this case we let s_B be .3, as against the .1 of case XI-b. All other propensities, including m_B, are the same as before. The investment in which country A can engage without permanent change in her trade balance, is now much less than before; to wit, it is $44.44 per period, one third of what it was in case XI-b. With this modest investment—modest in comparison with the $100 investment of country B—A's induced imports, in the beginning, will fall short of her foreign-induced exports.

Thus, a gold inflow or induced foreign lending will take place during the transition period.   But the export surpluses vanish rather quickly.

It is worth mentioning that in all cases of Model XI the ultimate volume of induced exports (= imports) in country A exceeds considerably (by 50 per cent) the volume of autonomous investment.   This is not surprising, for m_A exceeds

Period	COUNTRY A: S=0.2 M=0.3								COUNTRY B: S=0.3 M=0.15							
	K=5.00			Foreign Balance				Income from 2+5+7	K=3.33			Foreign Balance				Income from 10+13+15
	Spendable Income			=12	=-14				Spendable Income			=4	=-6			
	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Net Δ(X-M)	Auton ΔI	Current ΔY	Past ΔY	Current ΔC	Induced ΔS	Home-Ind ΔM	For-Ind ΔX	Net Δ(X-M)	Auton ΔI	Current ΔY
1							33.33	33.33							100.00	100.00
2	33.33	16.67	6.66	10.00	15.00	+5.00	33.33	65.00	100.00	55.00	30.00	15.00	10.00	-5.00	100.00	165.00
3	65.00	32.50	13.00	19.50	24.75	+5.25	33.33	90.58	165.00	90.75	49.50	24.75	19.50	-5.25	100.00	210.25
4	90.58	45.29	18.12	27.17	31.54	+4.37	33.33	110.16	210.25	115.64	63.07	31.54	27.17	-4.37	100.00	242.81
5	110.16	55.08	22.03	33.05	36.42	+3.37	33.33	124.83	242.81	133.55	72.84	36.42	33.05	-3.37	100.00	266.60
6	124.83	62.41	24.97	37.45	39.99	+2.54	33.33	135.73	266.60	146.63	79.98	39.99	37.45	-2.54	100.00	284.08
7	135.73	67.87	27.14	40.72	42.61	+1.89	33.33	143.81	284.08	156.25	85.22	42.61	40.72	-1.89	100.00	296.97
8	143.81	71.91	28.76	43.14	44.55	+1.41	33.33	149.79	296.97	163.33	89.09	44.55	43.14	-1.41	100.00	306.47
9	149.79	74.89	29.96	44.94	45.97	+1.03	33.33	154.19	306.47	168.56	91.94	45.97	44.94	-1.03	100.00	313.50
∞	166.67	83.34	33.33	50.00	50.00	0	33.33	166.67	333.33	183.33	100.00	50.00	50.00	0	100.00	333.33

TABLE XI-d.—Investments increased in A and in B with induced imports exactly balancing.

s_A (by 50 per cent); the induced imports are $m_A Y_{A,\infty}$, while $s_A Y_{A,\infty}$ must equal the autonomous investment.

One more case shall be taken up in order to show the effect of a change in B's marginal propensity to import.   We let m_B be .15, instead of the .2 of case XI-c, leaving all other propensities unchanged.   Since the imports of B, i.e., the exports of A, will now be smaller, A cannot engage in so much investment as before, if a continually unfavorable balance of payments is to be avoided.   Fixing the rate of investment at $33.33 per period, country A will have slight export surpluses during transition, and a zero balance of induced trade in equilibrium.

The multipliers are not affected by the change, except if one follows the misleading technique of treating exports indiscriminately as parts of the multiplicand. If this technique is used, the investment-plus-export multiplier of country B is 2.2 in case XI-d, as against 2 in case XI-c. The home-investment multiplier in B is in both cases 3.33. In country A, the home-investment multiplier is invariably 5 and the investment-plus-export multiplier is invariably 2.

A WORLD WITH TIME AND SEVERAL COUNTRIES

Models VIII, IX, X, and XI were all confined to two countries only, and the formulas that were furnished for the cases described by these models were all confined to ultimate equilibrium values only. Tenacious consistency would require that we pursue our problems further. We might develop our formulas to make them applicable to shorter intervals; that is to say, we might derive formulas which include time as a variable and, thus, give incomes as a function of time. And we might construct models, and obtain formulas, which include several countries; some of these countries could "actively" participate in the interplay in that their rates of domestic investment also change and other countries might merely react to the changes elsewhere and participate in the interplay through the repercussions which emanate from them.

I shall not submit my readers and myself to such an endurance test. The principles for the solution of these tasks were worked out in earlier chapters. The time element was dealt with when the formulas for Model IV were expanded to include time as a variable (see p. 87 above and Appendix A). The inclusion of several countries was effected in our Models V, VI, and VII, and the formulas were expanded to include the propensities prevailing in three countries (see p. 110 above and Appendix B).

Not that the tasks to include time and more countries in the formulas produced in Chapter VII and in this chapter would be particularly difficult. But I do not believe that our insight into the relevant relationships would be much enhanced by this extension of our analysis. Thus, I leave it to other students to work out the problems in question as exercises in algebra and economics.

Chapter X

APOLOGIES AND CONFESSIONS

EXACT FORMULAS FOR INCOMPLETE THEORIES

At the end of the preceding chapter I apologized for not introducing into the analysis of foreign-induced-trade multipliers and home-investment multipliers with foreign repercussions the same complications to which our analysis of the autonomous-foreign-trade multiplier had been subjected. Of course, there is always the question to what extent an analysis could be intensified without too seriously diminishing returns. Would it be worthwhile to continue relaxing and dropping simplifying assumptions? And in what direction should the addition of more special assumptions proceed? Or had I better apologize for having gone already farther than it is profitable to go?

Indeed, one may well ask what is the good of all the complications which we have introduced in some parts of the study, and of all the mathematical precision with which they were investigated and exhibited. After all, we know full well (1) that we are still neglecting a large number of important factors (changes in prices, interest rates, exchange rates, induced home investment, etc.); (2) that the factors which are taken into consideration are grossly and violently simplified (all countries of equal size, propensities uniform in all countries but the two immediately concerned, etc.); (3) that we labor with precise functions of variables the magnitudes of which are in fact unknown to us (marginal propensities to

save and to import in the various countries); and (4) that
these variables, even if their magnitudes were known to us
now, cannot be assumed to be constant over time. Must we
conclude from these or other considerations that our explora-
tions were a waste of time and effort? I submit that such a
conclusion would not be warranted and that the explorations
have thrown light upon a number of obscure points.

It may perhaps appear as a strange contrast to the admitted
omissions, simplifications, and treatment of ever-unknown and
never-stable variables as known and stable, that the various
models which we presented were made up of figures calculated
down to two or more decimal places. But, surely, the use of
arithmetically correct figures can hardly be of any harm in
these models. And these models, I believe, do have consider-
able usefulness in the demonstration of the working of forces
which undoubtedly are operating in reality. Whereas many
previous expositions of the multiplier principle confined them-
selves to the propensity to save as the sole coefficient, and
whereas even those expositions which gave consideration to
induced imports did not deal systematically with foreign reper-
cussions—foreign-induced changes in exports—the models
presented here show all these forces in combined operation;
they help in understanding the relevant processes and the
interlocking sequences of trade balances and incomes, chang-
ing through time.

We may not have the information enabling us to substitute
real numbers in our formulas and, thus, to "know" what, for
example, an additional meat export will do to Argentina's
income. But this is not the real purpose of the formulas.
Their purpose is to exhibit certain relationships between
independent and dependent variables, to show whether they
are positively or negatively correlated, to tell whether it is
their magnitudes or their proportions which matter, to indi-
cate which ones are more important and which less, and, last

but not least, to warn us about the things which we are to find out before we try to make general statements, not to speak of predictions.

THE QUANTITATIVE SIGNIFICANCE OF THE FOREIGN FACTORS

The question may be asked whether the two factors which complicate the investment and foreign-trade multipliers—induced imports and foreign repercussions—are important enough to bother with.   The considerable effects which they were shown to have in our examples may be due merely to the special assumptions upon which they were based.   For instance, we assumed almost invariably that the exporting country had a marginal propensity to import of .3.   This is probably higher than the import propensity actually is in large countries,[1] and our assumption may thus give an exaggerated impression of the significance of this factor.   Or, the assumption that there was only one country besides the exporting country (as in Model IV) showed the foreign repercussions especially high, much higher than they would be in reality with the large number of foreign countries over which the repercussions may be dissipated.

Although the whole theory of the foreign-trade multiplier, as long as it neglects reactions through price changes, interest

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[1] The following estimates or guesses of the marginal propensity to import were made or quoted by various authors: For England .17 (R. F. Kahn, "The relation of home investment to unemployment," *Economic Journal*, Vol. 41, 1931, p. 186; Colin Clark, *The Conditions of Economic Progress*, p. 479); for the United States, .073 (Imre de Vegh, "Imports and income in the United States and Canada," *Review of Economic Statistics*, Vol. 23, 1941, p. 133); for Canada .36 (de Vegh, *op. cit.* p. 136); for Australia .25 (Colin Clark and J. G. Crawford, *The National Income of Australia*, p. 100); Queensland .39 (Colin Clark, *op. cit.* p. 97); for Denmark .40 (J. Warming, "The financing of public works during depression," *Economic Journal*, Vol. 42, 1932, p. 217).

changes, foreign-exchange rate changes and foreign retaliatory measures, certainly implies an understatement rather than an overstatement of the probable repercussions, it must be admitted that the special assumptions made in the various examples may have magnified the role of the two income-retarding foreign-trade factors, i.e., the home-induced increase in imports and the foreign-induced reduction in exports. However, there appear to be good reasons for assuming that, if one of the two foreign factors should become negligible, the other will operate with considerable force.

If the exporting country is a relatively small country, the foreign repercussions that could hold down its income increase subsequent to an autonomous export increase are likely not to be severe. For, the exports of the small country represent probably only small portions of the total imports of other countries, and the induced reductions of imports of these other countries will probably only to a negligible extent hit back upon the small country which had been favored by the autonomous export increase. On the other hand, the induced imports of the small exporting country are likely to pick up considerably, because small countries are apt to have a high marginal propensity to import. Thus, the leakage through imports will be high.

If the country which gains an autonomous increase in exports is a relatively large country, induced imports will usually not constitute a severe leakage. For, large countries are likely to have low marginal propensities to import (because most of the increased consumption can be supplied by the home industry). On the other hand, the exports of a large country are likely to represent large portions of the imports of other countries, and the induced reduction in imports of these other countries will probably to a considerable extent constitute imports from the large country which has first had the autonomous increase in exports. Thus, the large country is

likely to feel heavily the repercussions from the reduction in foreign buying power.[1]

Hence, although it is conceivable that the foreign factors are of little significance in particular cases, the presumption is that their combined effect is far from negligible.

THE ASSUMPTION OF STABLE PRICES

We must not leave our analysis before we make peace with some of the most dangerous among the assumptions upon which the whole theoretical structure was built. How badly are our results threatened by the induced price changes and the price-induced changes in foreign trade? By keeping these strong forces temporarily quiet we have avoided fighting on two fronts at a time. But can we expect that our deal with income changes and income effects will be permanently safe from interference by price changes and price effects?

Since changes in incomes occur in all countries concerned and, indeed, are the crux of multiplier theory, product prices can be unchanged only if the supplies of all goods and services are perfectly elastic. When there is unemployment of every sort of human, natural and man-made resources, when pro-

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[1] These statements are in full agreement with those which William A. Salant makes in his essay, previously cited, (*Public Policy*, Vol. II, p. 220) about the "greater monetary autonomy" of a large country, that is, of a country "whose imports are only a small portion of its own national income, but whose exports are a large fraction of world exports." Mr. Salant, discussing the effects of domestic expansion (home investment) upon the balance of payments, states: "In general, we may say that a country may pursue an independent monetary policy without regard to the balance of payments if foreign trade plays a small part in its economy, but its foreign trade is a sizeable portion of world trade." In this case, the smallness of the marginal propensity to import and the largeness of the foreign repercussion factor work in the same direction: the former makes imports rise only a little in consequence of the domestic income expansion, the latter makes exports rise—and thus offset a part of the small adverse change in the balance of payments—because foreign incomes will be increased through the additional imports of the expanding country and will result in increased purchases from it.

ductive capacities are utilized well below capacity, some degree of variation in the quantities supplied will be possible without any price changes. Yet, even in the darkest depression, there will be numerous products of which larger quantities will be forthcoming only at advanced prices and, likewise, of which prices can still further recede if demands fall off. Thus, there will always be some prices that are liable to rise in the country whose income expands, and liable to fall in the country whose income shrinks. Among these prices will probably be some which, directly or indirectly, influence the import demands or export possibilities of the countries concerned.

The direct influences upon exports are obvious: if prices of export articles rise, exports will fall in physical quantity and may fall in total value; if prices of export articles fall, exports will rise in physical quantity and may rise in total value. Indirect influences, repercussions upon foreign trade via changes in purely domestic prices, are less obvious but easily explained. Assume, for example, that prices of some purely domestic (home-produced and home-consumed) commodities rise when effective demand expands in the country which is favored by an autonomously improved trade balance. These price advances may in two ways impinge on foreign trade. Through cost-relationship with the domestic commodities, the production cost of export articles may be increased. And through demand-relationship with the domestic commodities, domestic demand for import articles may be increased. The former link, the higher cost of making export articles, may result in a price-induced fall in exports. The latter link, when the higher prices of domestic goods turn buyers to imported substitutes, will result in a price-induced increase in imports. And, of course, the opposite tendencies are likely to be in operation in the country which has experienced the adverse change in the trade balance.

The theory of the autonomous-foreign-trade multiplier stated that an autonomous "disturbance" of the trade balance would lead merely to a part-way rectification (and only if there were no induced changes in saving, to a complete rectification) of the balance via permanent income changes in the countries concerned. Now we have found that the price effects upon foreign trade are all likely to work in the same direction, that is, also toward a rectification of the trade balance. This will mean that the rectification will be more complete than if income effects alone had to do the job; but it will mean also that incomes will have less of a chance to be permanently lifted or reduced from their former level by exactly those amounts which the multipliers left us to expect.[1] There is no *a priori* reason for the price-induced changes to be a mild or a strong counterforce to the original stimuli. Their strength will depend on an unpredictable scramble among a host of income-elasticities and price-elasticities of demand and supply.

There are first the income-elasticities of demand; they would tell of which goods larger or smaller amounts will be demanded when incomes rise or fall. There are next the price-elasticities of supply; they would tell of which goods larger or smaller amounts will be produced without changes in price, and of which goods prices will advance or recede when production volumes adjust themselves to changed market situations. Then come the elasticities of derived demand for intermediate products and factors of production, and the elasticities of their

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[1] We have confined our remarks here to the price effects upon exports and imports, but should mention also the price effects upon saving. The controversial question whether real income or money income is the basis of the marginal propensity to save is irrelevant as long as prices are assumed to be stable. But when prices change along with incomes it will make a difference whether savers are induced by changes in money income or by changes in real income. (All this is still further complicated by shifts in the distribution of money income and real income.) I am not equipped to deal with these problems at this time.

supplies; they would tell about positive or negative or zero changes in certain production costs and this, in turn, would tell which cost curves will be shifted and how supply conditions of several other products will be affected. At the same time, there are the cross-elasticities of demand; they would tell of which goods larger or smaller amounts will be demanded when the prices of other competing or completing goods rise or fall. And then would come the question which of all these goods were export goods, which potential export goods, which goods competing with imported ones, etc., etc.

The advantages of provisionally ruling out all these complications by the stable-price assumption are certainly great. For there is little that a general theory can do about this mess of "possibilities." The only generalizations that can safely be made are these: in depression times and in more monopolistic industries supply curves are probably more nearly of perfect elasticity than in prosperity and in more competitive industries. It is quite unlikely that all prices will remain stable in the face of changes in effective demand; but it is quite possible that the number of prices that will significantly change under the impact of changing demand when there is unemployment of men, machines and money, is small; perhaps small enough that the theory of the income multiplier can be applied without too many qualifications—provided that all is quiet on the labor front.

THE ASSUMPTION OF STABLE WAGE RATES

The existence of unemployment is often regarded as a sufficient justification for accepting (more than provisionally) the assumption of stable wage rates. That wage rates will not easily fall below a certain level even if a bad depression gets still worse, seems to be in correspondence with our experience. But that the existence of unemployment should prevent wage rates from rising in the face of an increase in effective demand,

is not confirmed by the experience of recent years. That many unemployed laborers would be willing or even anxious to take jobs at prevailing wage rates does not mean that these wage rates actually would be left unchanged once jobs begin to open up. In a world in which trade unions, more interested in higher wage rates than in fuller employment, battle for a raise at the first signs of better business, the inference from unemployment to wage stability is a *non-sequitur*.[1]

The bearing of changing wage rates upon the theory of the multiplier is paramount. Rising money income with constant wage rates is one thing. Rising money income with rising wage rates is another.[2] The rise in wage rates may affect costs and prices, investments, exports, imports, employment, incomes. However, these effects cannot be built into multiplier theory. Apart from the complexity of the "possibilities" which are connected with "given" wage rate changes—can these changes be taken as given if they are themselves income-induced? And assumptions about the comparative strength of different unions, and about their attitudes, are certainly beyond the scope of any general theory.

The moral of all this is not that the assumption of stable wage rates, in analyses such as we have been engaged in, should have never been made. To the contrary, the assumption of wage rates which change in uncertain ways and to an uncertain extent would make the theory hopelessly

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[1] On this point see the lucid analysis by Joan Robinson, *Essays in the Theory of Employment*, London, 1937, pp. 7 ff. and 25 ff., especially p. 9: "There is a certain level of employment, determined by the general strategical position of the Trade Unions, at which money wages rise. . . . "

[2] Lord Keynes evaded most of the difficulties involved by expressing all magnitudes in terms of wage units rather than in terms of money. If a rise in wage rates left aggregate investment in terms of money unchanged, aggregate investment in terms of wage units would be reduced. Thus, the effects of the wage increase—just as all other repercussions—would be shoved back into the multiplicand. A theory of the money-income multiplier, however, must deal with the problems explicitly.

cumbersome. As long as employment is apt to vary more heavily than wage rates, an orderly procedure had better start with the assumption of fixed wage rates. But one should never even for a minute forget that the assumption was made, and must never dare to apply the theory without making explicit reservations concerning possible movements of wage rates.

If the flexibility of wage rates were so perfect that the price of labor would be the changing and employment the constant factor, then, indeed, multiplier theory would be rather useless. The ("truly inflationary") expansion of the money flow through autonomous improvements of the trade balance or through an increase in money disbursements for home investment, would not be interesting enough to require the apparatus of multiplier analysis. We would "know"—for it would follow from the assumption of unchanged employment—that, with given real home investment, an improvement in the trade balance could never expand consumption in real terms but could only reduce it; and that, if both investment and consumption in real terms remained unchanged, an autonomous favorable change in the trade balance would necessarily be completely rectified by induced adverse changes.

This simple reasoning explains why an analysis based on the assumption of full employment must be more, or almost exclusively, interested in relative price changes and the barter terms of trade. An improvement in the barter terms of trade would be the only way of increasing, with a given state of the arts, the real national income. And it becomes very clear why the "automatic" rectification of the trade balance is claimed to be so speedy in an economy with flexible wage rates and unchanged employment. As we have just said, increased exports or reduced imports could be sustained only if factors of production were successfully bidden away from home investment or consumption—a tug-of-war which might be only

slightly relieved by improved terms of trade, or possibly slightly aggravated by less favorable terms of trade. Of course, both home investment and consumption would struggle hard to hold their own in the markets of factors and products. It would be most unlikely that they should lose out; as a rule, they would quickly succeed in fencing off the encroachment by the "favorable" (!!) trade balance. Now, it is largely a matter of the actual situation on the labor front which of the two lines of analysis is more applicable. We may, more often than not, be far away from "full employment"; but this is not the essential point. The real issue is the upward-flexibility of wage rates, which may come into operation long before anything approaching full employment is attained. An aggressive labor movement can always create conditions which would make the assumptions of full-employment analysis applicable to an under-employment economy. Only as long as employment is appreciably more variable than wage rates, will the models and formulas of multiplier theory, supplemented by adequate qualifications concerning the degree of wage flexibility, be an appropriate instrument of analysis.

THE ASSUMPTION OF STABLE INTEREST RATES

The changes in the level of investment, the changes in trade balances and foreign lending, and the changes in induced saving, all may imply, as was pointed out above in several places, significant changes in the reserve position of banks. Most of our models reached "equilibrium positions" which, paradoxically, involved continuing gains or losses of gold or foreign balances. Thus, they were equilibrium positions only if the reserve positions of the banks either did not matter or were deliberately kept unchanged by the monetary authorities. Otherwise, the failure to take account of the effects which induced changes in the reserve positions of banks are liable to

have upon the banks' interest and lending policies, may seriously reduce the applicability of the whole analysis; it certainly reduces it to mere short-run theory .

The neglect of these matters goes back to the Keynesian idea of the perfectly (or almost perfectly) elastic lower end of the liquidity preference schedule and to the habit of regarding interest rates as independent variables.[1] To be sure, in the United States and England during the years of the depression the supply of bank funds seemed actually to be perfectly elastic in both directions, the banks being willing to use all those outlets for loan funds which they could find without much change in the terms of lending. But it would be a mistake to generalize these special conditions. A "general theory" cannot be based on the assumption of stable interest rates.

A "general theory" may perhaps assume that the liquidity preferences of individuals, firms and banks, and the policy preferences of the monetary authorities, are "given," but not a stable level of interest rates. However, the job of theorizing would be rather horrible. And the results would hardly be worth the sacrifice of simplicity. The effects of changing bank reserves and interest rates can much more easily be taken care of by way of subsequent qualification to an analysis which has first neglected them.

The effects of changed interest rates upon our analysis are at least threefold: (1) through changes in commodity prices, particularly prices of durable goods; (2) through changes in the propensity to save; (3) through changes in the rate of investment. We need not add anything here concerning changes in prices; all that can be said in this context has probably been said above. Nor can we say much concerning interest-effects on the propensity to save. The effects of the interest

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[1] See on this point Oscar Lange, "The rate of interest and the optimum propensity to consume," *Economica*, New Series, Vol. 5, 1936, pp. 12 ff.

rate upon the level of saving are not very great, if we can believe the authorities on this question. But in any case, the mistake of neglecting this source of disturbance does not make matters much worse; for there are probably more potent influences upon the propensity to save which were all ruled out in the theory of the multiplier. To include them would make it more complicated, without altering the underlying principles. We must, however, comment on the third of the three types of interest effects, the interest-induced changes in investment.

We have chosen, in this analysis, to rule out all sorts of induced changes in home investments. One might have included export-induced investments and, above all, income-induced investments. Very important work has been done on the interrelationship between investment-generated income and income-induced investment, that is, on the interactions between the multiplier and acceleration principles.[1] Some writers on multiplier theory deemed it desirable to make assumptions about stable marginal propensities to invest.[2] For several reasons, I have not followed their example. First of all, while I persuaded myself, with many misgivings, to accept stable marginal propensities to save as provisional working hypothesis, I cannot yet swallow the assumption of stable marginal propensities to invest. Secondly, if one did assume stable marginal propensities to invest, the results in terms of money income would be the same as if one had assumed correspondingly higher marginal propensities to

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[1] Paul A. Samuelson, "Interactions between the multiplier analysis and the principle of acceleration," *Review of Economic Statistics*, Vol. 21, 1939, pp. 75 ff.

[2] The marginal propensity to invest and the marginal propensity to consume together constitute the marginal propensity to spend. To "substitute for Keynes's 'marginal propensity to consume' the 'marginal propensity to *spend*'" was suggested by Professor James W. Angell, *Investment and Business Cycles*, New York, 1941, p. 195. His suggestion was accepted by Lloyd A. Metzler, *loc. cit;* see above pp. 22 and 184 n.

consume; hence no new principle would be involved in the analysis. By assuming, for the general multiplier models, that income-induced changes in investments are zero, one does not bar the way to introducing such changes, at any point of the analysis, in any magnitudes which they may be expected to have under particular circumstances.

I submit that the same holds true concerning interest-induced changes in investment. To rule them out in multiplier analysis, does not mean to deny their possible or probable occurrence. That a model is constructed to show what happens to foreign trade and national income *if and as long as* the level of home investment is stepped up by a certain amount, does not mean that home investment can permanently be kept at that level. If induced changes in the reserve position of the banks force them to change their terms of lending and, thus, bring about a change in the investment level, one can introduce this interest-induced change at the point at which it is expected to arise and in the magnitude which it is expected to have. The reserve positions of banks, particularly since they may at any time be controlled by the policy of the monetary authorities, cannot conveniently be made a determining factor in general multiplier analysis, although they must, of course, be a factor not to be neglected when multiplier analysis is applied to a concrete situation.

THE NEO-MERCANTILIST INTERPRETATION OF THE THEORY

Multiplier theory has become fashionable not merely as a technique of analysis but also as a technique of persuasion in the advocacy of economic policies. Multiplier theory is being used in the endorsement of all sorts of programs, ranging from taxation policies for the sake of greater multipliers to lending and spending policies for the sake of greater multiplicands. It is not surprising that the theory of the foreign-trade multi-

plier has also come to play a major role in the support of commercial policies.

No matter how large the leakages are which result from induced saving, induced imports and foreign-induced export reductions, the foreign-trade multiplier is always likely to be greater than zero and usually greater than one. This means that, under the assumptions of the expounded theory, an autonomous improvement of the foreign-trade balance will generate income, not only temporarily but "permanently." Impressed by the principles of the theory more than by its underlying assumptions, neo-mercantilists are quick in appropriating the new tenet to their stock of arguments for protective trade restrictions. Exports gained through subsidies, and imports cut down through tariffs, quotas or exchange restrictions, are recommended as efficient generators of national income.

The protectionistic interpretation of the theory of the foreign-trade multiplier, I believe, is illegitimate in some respects, short-sighted in others, and rather unfortunate as a whole. It is illegitimate if it "applies" the theory without ascertaining the appropriateness of the hypothetical assumptions—especially those concerning price and wage stability—under actual circumstances. After all, if it is income expansion that is sought, one must make sure that possible price effects are reduced to a minimum. The burden of the proof that absolute and relative price increases would under the actual circumstances be contrary to all reasonable expectations, rests on the advocates of the protectionist policy. Surely, this proof is not easily produced; but usually it is not even offered.

The short-sightedness of the protectionistic exploitation of the theory of the foreign-trade multiplier is particularly serious with respect to the likelihood of retaliations on the part of other countries. One should think that everybody has learnt by now that not even temporary relief can be expected from

added protection, because other nations are too quick in taking retaliatory measures. This beggar-my-neighbor game among nations is as childish as it is cruel, for, tragicomically, in this game nobody wins and everybody loses in the long run.[1] In terms of money income, of course, not everybody must lose; after a frantic exchange of blows one nation may have experienced more favorable than adverse changes in its trade balance. But in terms of real income it is almost inconceivable that a nation should come out of the scuffle without loss.

Of course, it was always "the other one" who has started it, and how could anybody put up with it and refrain from defense? Thus the question becomes one of merely "defensive" measures (though a defense which hurts the others is really a counterattack). But even as mere defense measures, tariff increases and other trade restrictions are usually inane and wasteful. If the purpose is no other than to avoid an income decline which has started from an autonomous export reduction due to protectionism abroad, there are cheaper and more effective ways of achieving it. (This is so at least with regard to nations which are not suffering from an acute lack of gold or foreign reserves.) It is inexcusable if among the various possibilities of expanding primary disbursements, or of offsetting their contraction, the one method is chosen which causes a contraction in other countries and, at the same time, reduces the international division of labor all around. An autonomous improvement of the trade balance is an autonomous deterioration for one or more other countries. Hence it is likely to cause these other countries to take "the next move," which, while it reverses the change in the trade balance, continues in the same deleterious direction: toward degeneration of international relations and destruction of the international division of labor.

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[1] *Cf.* Joan Robinson, *op. cit.* p. 211.

If full account is taken of all possible short-run effects—the (economically determined) changes in relative prices as well as the (politically determined) retaliatory measures—and also of the fairly certain long-run effects—impaired productivity through reduced division of labor—then the theory of the foreign-trade multiplier cannot be claimed as a tenable argument for protection.

A CONSTRUCTIVE LESSON

If people insist on "deriving" positive policy recommendations from economic theories however incomplete—a practice which this writer has no wish to encourage—they might at least be sensible enough to turn from destructive commercial policy to more constructive "applications" of multiplier theory in international trade matters. For example the case of parallel expansion in the major countries of the world may, under certain circumstances, have much to be said for it. I believe, however, that it is the case of capital export from countries with abundant capital supply to countries with meagre capital supply which is probably the most significant "lesson" from our analysis. Its importance for the period of reconstruction after the world war can hardly be overestimated.

An autonomous capital export, we saw in Chapter VIII, need not cause a reduction of primary disbursements in the lending country if the supply of capital in the country is large and if its money market is in a liquid position. An autonomous capital import, on the other hand, in a country with scarce supply and urgent demand for funds, is very likely to produce there a brisk expansion of disbursements. As a result, both nations will have their incomes increased, the borrowing country in consequence of the increased investment activity, the lending country in consequence of the increased export business. These exports and their income-generating effects may even overcompensate some slight reductions in

home investment in the lending country if such could not be avoided in the course of the capital outflows. (See, for example, cases X-c and X-e, discussed in Chapter IX.)

In most treatises on international capital movements a contraction or deflation in the capital exporting country is taken almost for granted. The foreign assets are usually seen as competing with domestic capital assets, if not with domestic consumption expenditures. Now it has become clear (largely under the influence of Lord Keynes' teachings) that foreign assets, at times, might compete only with liquid funds; the acquisition of the foreign assets involves then merely parting with liquidity, leaving home investment and consumption for the moment unaffected. The real activation of the liquid funds ("dishoarded" or "newly created")—that is to say, their injection into the income flow—takes place not immediately upon their use for the foreign investment but only when the export business begins to pick up owing to the foreign expansion. It is through this effect on exports that spontaneous foreign lending, instead of being a "draw" on domestic disbursements and incomes, becomes a generator of national income.

If domestic investment opportunities (at institutionally possible interest rates) are relatively poor, and if the savings propensity in the nation is relatively high, the export business which is induced by the expansion abroad (which, in turn, is made possible and evoked by the capital flow) may become a "life-saver" of a certain level of domestic activity and national income. In this instance, the national income would not gain a net increase; but an otherwise threatening decline may be staved off through the loans to and investments in foreign countries.

The energizing (income-creating or income-maintaining) effects of spontaneous foreign lending and foreign investment are not likely to be disturbed or obviated by price effects.

Increases of prices abroad, if such occur in the course of the expansion, can only reinforce, not weaken, the rise in the exports of the lending country.   For, clearly, both income effects and price effects would operate in the same direction: higher prices abroad would enlarge further the import demand already swelled-up through higher incomes.   In the lending country price changes are not likely to arise if the exports are merely income-maintaining; if the exports are actually income-expanding, price advances may occur and possibly check the rise in exports.   But, of course, the rise in exports cannot be altogether prevented by a rise in prices in the exporting country, if this rise in prices is the very result of the increased exports.

The use of multiplier theory as an argument for generous foreign lending—the "imperialistic" interpretation of the theory of the multiplier—probably will be vehemently opposed by advocates of economic and political isolationism.   They may ask: If increased exports are hailed because they permit a fuller utilization of our productive resources, why not utilize these resources for domestic improvements?   Why should we build up the productive equipment of backward nations when there are plenty of things to do at home?   There may not be enough profitable investment outlets at home, but are slum clearances, road ameliorations, improvements of educational and recreational facilities, etc., in our own country not more important to us than all sorts of projects in China or India or the reconstruction of Europe and Russia?

These questions cannot be answered on grounds of economic theory.   What the economist can do is to call attention to a number of things which should not be disregarded when the issues are weighed and the case of "more domestic ameliorations versus loans to foreign nations" is under advisement.   It goes without saying that the foreign loans are not suggested to take the place of all domestic amelioration projects; there is

room for both. But the questions asked by the opponents of "imperialistic" lending seem to militate against all foreign loans, suggesting that we give none and have instead an extra dose of domestic amelioration. The following are among the points which seem to be significant in this connection.

(1) If the foreign lending is done, in part at least, by private capital, whereas domestic amelioration projects are a matter of government finance, serious problems of fiscal policy would be avoided or alleviated by the one, created or aggravated by the other alternative. (2) Apart from the direct effects of fiscal policies, the political implications and the effects upon business psychology are highly important. With respect to domestic investment opportunities and private investment activity, increased government expenditure may be a retarding factor, while increased export business, resulting from the foreign lending, may be an energizing factor. (3) The effects of the loans to foreign nations upon the living standards of peoples abroad may be enormous, whereas the extra dose of domestic ameliorations might be a minor element in the living standard of our people. (4) The development of the foreign economies which is made possible through our loans may create new opportunities for international division of labor and thus, in the long run, actually increase the productivity of our labor. (5) While the extra dose of domestic ameliorations is not profitable and perhaps not even very productive, the loans to foreign nations, if adequately administered, may earn interest for the lending nation. (6) While economic isolationism creates envy and resentment abroad and breeds international conflict, increased prosperity abroad, facilitated by our loans, and the close ties created by them, may contribute to satisfactory international relations and to the maintenance of peace. Almost every one of these six points contains a hint of a considerable long-run contribution which a policy of generous foreign lending can make to the

economic welfare of the lending nation, apart from the short-run contribution to income which was the topic of the multiplier theory expounded in previous chapters.

None of the six points, however, has any immediate connection with the theoretical analysis to which this book was devoted. Nor was any one of them tested and proved here for correctness or relevance. Which amounts to the confession that the policy recommendation has not naturally grown out of the body of economic analysis contained in these pages, but was rather arbitrarily appended to it. This, however, is not a special weakness of the policy which was favored here. It is common more or less to all policy recommendations. For, while economics can well serve to criticize policies, by showing that the reasoning on which they are based is faulty, inconsistent or woefully incomplete, economic analysis will rarely be complete enough to permit unambiguous conclusions from it to be drawn in the form of definite and positive prescriptions for the governments of the nations. This remark does not mean to belittle economic theory but merely urges caution in its application. In expressing this warning, the theorist does not abdicate but, on the contrary, calls for more and still more analysis.

If the practical significance of economic analysis is not measured in terms of immediate applicability to policy making but, instead, in terms of improved insight and better understanding of complex relationships, we need not be gloomy about the prospective yields of the effort which we have invested in our study.

Appendix A

THE MULTIPLIER AS A FUNCTION OF TIME, WITH ONLY TWO COUNTRIES INVOLVED

The task here is to develop a formula[1] which gives us the increments of income (generated by an autonomous increase in export) not after an infinite time interval but after t periods have elapsed.

The assumptions are those ruling the analysis in the text, in general, and of Model IV, in particular. Hence, only two countries, A and B, are involved and the marginal propensities to save and to import are given and constant.

For the sake of convenience we repeat the notations:

$X,$ the autonomous increase in exports from A to B;

$t,$ the number of time periods elapsed, counted from the first occurrence of the additional export X;

$Y_{A,t},$ the increment in income received in A during the tth period counted from the first occurrence of the additional export X;

---

[1] In the preparation of this appendix I have received help from able mathematicians. I am taking this opportunity to express my gratitude to Professor Harry M. Gehman who has given generously of his time and has guided me along a path of reasoning which might have been impassable for me alone. Much credit is due also to my research assistant, Mr. Joseph Ullman, who has acted as deputy guide in terrain in which I do not feel yet quite at home. Professor Paul A. Samuelson was kind enough to check the findings and to console me that there is no simpler form to which the problem might be reduced.

$Y_{A,t-1}$,   the increment in income received in A during the $(t-1)$th period;

$Y_{A,t-2}$,   the same as above, but for the $(t-2)$th period;

$Y_{B,t-1}$,   the increment in income received (usually negative) in B during the $(t-1)$th period;

$Y_{B,t-2}$,   the same as above, but for the $(t-2)$th period;

c_A,   the marginal propensity to consume home-produced goods and services in country A; it is defined as the proportion of an increment of income (received in any period $t-1$) which is used (in period t) for additional consumption of domestic goods and services;

c_B,   the marginal propensity to consume home-produced goods and services in country B;

m_A,   the marginal propensity to import in country A; it is defined as the proportion of an increment of income (received in any period $t-1$) which is used (in period t) for additional imports;

m_B,   the marginal propensity to import in country B;

s_A,   the marginal propensity to save in country A; it is defined as the proportion of an increment of income (received in any period $t-1$) which is not used (in period t) for additional consumption or imports;

s_B,   the marginal propensity to save in country B.

Further notations will be introduced in the course of the argument.

We begin with developing formulas showing the recursions to immediately preceding periods.

In any period $t-1$, the income increment (as compared with period 0) in country A is the net result of the autonomous export, X, the propagated portion of A's income increment of the preceding period, $c_A Y_{A,t-2}$, and the foreign-induced reduction of exports, which, of course, is that portion of B's income decrement of the preceding period which is withdrawn from

purchasing imported goods, that is $m_B Y_{B,t-2}$.   Thus we write

(11) $Y_{A,t-1} = X + c_A Y_{A,t-2} + m_B Y_{B,t-2}.$

(The last term is negative because $Y_{B,t-2}$ is negative.)

This can be arranged to read

(11a) $Y_{B,t-2} = \dfrac{1}{m_B} (-X + Y_{A,t-1} - c_A Y_{A,t-2}).$

In the period $t-1$, the income decrement (as compared with period 0) in country B is the net result of the autonomous import, $-X$, the propagated portion of B's income decrement of the preceding period, $c_B Y_{B,t-2}$, and (as a partial offset) the foreign-induced increase in B's exports, which, of course, is that portion of A's income increment of the preceding period which is used to purchase imported goods, that is, $m_A Y_{A,t-2}$. Thus we write

(12) $Y_{B,t-1} = -X + c_B Y_{B,t-2} + m_A Y_{A,t-2}.$

(The middle term on the right side of this equation is negative because $Y_{B,t-2}$ is negative.)

Substituting (11a) for the respective part of (12) we obtain

(12a)

$$Y_{B,t-1} = -X + \frac{c_B}{m_B}(-X + Y_{A,t-1} - c_A Y_{A,t-2}) + m_A Y_{A,t-2}.$$

Now that we know, through equations (11) and (12a), the income changes in A and B in period $t-1$, we can write the equation for A's income increment in the period t.

(13) $Y_{A,t} = X + c_A Y_{A,t-1} + m_B Y_{B,t-1}.$

By substituting (12a) we obtain

(13a) $Y_{A,t} = X + c_A Y_{A,t-1}$
$$+ m_B \left[-X + \frac{c_B}{m_B}(-X_A + Y_{A,t-1} - c_A Y_{A,t-2}) + m_A Y_{A,t-2} \right].$$

Multiplying out, the equation reads

$$(13b) \quad Y_{A,t} = X + c_A Y_{A,t-1} - m_B X - c_B X \\ + c_B Y_{A,t-1} - c_A c_B Y_{A,t-2} + m_A m_B Y_{A,t-2}.$$

By regrouping this becomes

$$(13c) \quad Y_{A,t} = X(1 - c_B - m_B) + Y_{A,t-1}(c_A + c_B) \\ + Y_{A,t-2}(m_A m_B - c_A c_B).$$

In equation (13c) income figures of country B have been eliminated. Of course, the effect of changes in B-income is still operating, but it is no longer explicit in the equation. Instead, the income increment of A in period t is expressed in terms of A's income increments in the two immediately preceding periods. Equation (13c) represents, therefore, a recursion formula, in which $(c_A + c_B)$ operates on the income increment of period $t - 1$, and $(m_A m_B - c_A c_B)$ on the income increment of period $t - 2$. It will be convenient to denote these two magnitudes by shorter symbols, let us say r and ρ. Thus,

$$(14) \qquad\qquad r = c_A + c_B,$$

which is the sum of the marginal propensities to consume home-made products in the two countries, and

$$(15) \qquad\qquad \rho = m_A m_B - c_A c_B,$$

which is the difference between the coefficient combining the propensities to import each other's products and the coefficient combining the propensities of both countries to consume their own products. In abbreviated notation, equation (13c), our recursion formula, becomes

$$(16) \qquad\qquad Y_{A,t} = s_B X + r Y_{A,t-1} + \rho Y_{A,t-2}.$$

We always know the values of the income increments for period 0 and period 1; for, $Y_{A,0} = 0$ and $Y_{A,1} = X$. Since

we can always compute the income increment for the next period, we can derive, by this laborious method, the income increment of period t. Using, however, the summation formula for geometric progressions, one can obtain an equation for the income increase of period t, an equation which is unwieldy and complicated even if further abbreviations are introduced. By way of introducing new notations for certain combinations of r and ρ which would appear repeatedly in the summation of the series, we call

$$(17) \qquad \alpha = \frac{r + \sqrt{r^2 + 4\rho}}{2}$$

and

$$(18) \qquad \beta = \frac{r - \sqrt{r^2 + 4\rho}}{2}.$$

Then, by a process the steps of which cannot be reproduced here,[1] the following equation can be obtained:

$$(19) \quad \Upsilon_{A,t} = X\left[\frac{s_B}{(1 - \alpha)(1 - \beta)} + \frac{(1 - \alpha - s_B)}{(1 - \alpha)(\alpha - \beta)} \alpha^t - \frac{(1 - \beta - s_B)}{(\alpha - \beta)(1 - \beta)} \beta^t \right].$$

The length of this forbidding-looking formula may give a wrong impression regarding the irksomeness of its use. As a matter of fact, the various combinations in which the few variables are used do not call for much labor or skill in calculation if absolute numbers are substituted. The basic variables are still m_A, s_A and m_B and s_B, with the t, for which the income is sought, appearing twice as exponent.

---

[1] The steps by which equation (19) has been derived were worked out for me by Professor Gehman. I was advised that an analogous development can be found in J. Tinbergen, *Business Cycles in the United States of America*, 1919–1932, League of Nations, Geneva, 1939, p. 144.

Appendix B

THE MULTIPLIER FORMULA INVOLVING PROPENSITIES IN THREE COUNTRIES

The task here is to develop a formula which gives the increments of income (generated or destroyed by an autonomous increase in export from country A to country B) after full equilibrium is reestablished.

The assumptions are those ruling the analysis in the text, in general, and of Model VII, in particular. Thus, several countries are involved here: A, the country with the additional export (X); B, the country which imports this additional export of A; and n other countries, which are all assumed to be exactly alike in every respect and which are therefore fully described by a description of one of them, country C. All countries involved $(A, B$ and $nC)$ are of equal weight in any one country's marginal propensity to import goods from other countries.

Given are X, s_A, m_A, s_B, m_B, s_C, m_C, and n. Wanted are $Y_{A,\infty}$ and $Y_{B,\infty}$, the increments or decrements of income in A and B which result from X after full equilibrium is established $(t = \infty)$; or, which is the same thing, k_A and k_B, the foreign-trade multipliers, if $Y_{A,\infty} = k_A X$ and $Y_{B,\infty} = -k_B X$.

Condition for equilibrium in each country is that increments in saving must be equal to increments in the export surplus of the country. The positive or negative export surplus will consist of autonomous as well as induced changes in exports and imports to and from all other countries.

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Thus, the equilibrium conditions in A, B and C, respectively, are expressed by the following three equations (see text, p. 112):

(24) $s_A Y_{A,\infty} = X - m_A Y_{A,\infty} + \dfrac{1}{n+1} m_B Y_{B,\infty} + \dfrac{n}{n+1} m_C Y_{C,\infty};$

(25) $s_B Y_{B,\infty} = -X - m_B Y_{B,\infty} + \dfrac{1}{n+1} m_A Y_{A,\infty}$

$$+ \dfrac{n}{n+1} m_C Y_{C,\infty};$$

(26) $s_C Y_{C,\infty} = \dfrac{1}{n+1} m_A Y_{A,\infty} + \dfrac{1}{n+1} m_B Y_{B,\infty}$

$$+ \dfrac{n-1}{n+1} m_C Y_{C,\infty} - m_C Y_{C,\infty}.$$

Writing $k_A X$ and $-k_B X$ for $Y_{A,\infty}$ and $Y_{B,\infty}$, respectively, we rewrite

(24a) $s_A k_A X = X - m_A k_A X - \dfrac{1}{n+1} m_B k_B X$

$$+ \dfrac{n}{n+1} m_C Y_{C,\infty};$$

(25a) $-s_B k_B X = -X + m_B k_B X + \dfrac{1}{n+1} m_A k_A X$

$$+ \dfrac{n}{n+1} m_C Y_{C,\infty};$$

(26a) $s_C Y_{C,\infty} = \dfrac{1}{n+1} m_A k_A X - \dfrac{1}{n+1} m_B k_B X$

$$+ \dfrac{n-1}{n+1} m_C Y_{C,\infty} - m_C Y_{C,\infty}.$$

We rearrange (26a) to

(26b) $Y_{C,\infty} \left[s_C + m_C \left(1 - \dfrac{n-1}{n+1} \right) \right]$

$$= X \dfrac{1}{n+1} (m_A k_A - m_B k_B),$$

and obtain

$$(26c) \quad \Upsilon_{c,\infty} = X \frac{1}{n+1} \frac{m_A k_A - m_B k_B}{s_c + m_c \left(1 - \dfrac{n-1}{n+1}\right)}$$

$$= X \frac{m_A k_A - m_B k_B}{s_c(n+1) + 2m_c}.$$

Substituting $(26c)$ in $(24a)$ and rearranging, we can write

$$(27) \quad s_A k_A X + m_A k_A X = X - \frac{1}{n+1} m_B k_B X$$

$$+ \frac{n}{n+1} m_c X \frac{m_A k_A - m_B k_B}{s_c(n-1) + 2m_c}.$$

Cancelling the Xs and reorganizing the last term in order to prepare for the transportation of the k_A to the left side, we have

$$(27a) \quad k_A(s_A + m_A) = 1 - \frac{1}{n+1} m_B k_B$$

$$+ \frac{n}{n+1} m_c \frac{m_A k_A}{s_c(n+1) + 2m_c} - \frac{n}{n+1} m_c \frac{m_B k_B}{s_c(n+1) + 2m_c}.$$

After the transport and further rearrangement this becomes

$$(27b) \quad k_A \left[s_A + m_A - m_A \frac{n}{n+1} \frac{m_c}{s_c(n+1) + 2m_c} \right]$$

$$= 1 - \frac{1}{n+1} m_B k_B - \frac{n}{n+1} m_B k_B \frac{m_c}{s_c(n+1) + 2m_c}$$

and finally,

$$(27c) \quad k_A = \frac{1 - m_B k_B \left[\dfrac{1}{n+1} + \dfrac{n}{n+1} \dfrac{m_c}{s_c(n+1) + 2m_c} \right]}{s_A + m_A \left[1 - \dfrac{n}{n+1} \dfrac{m_c}{s_c(n+1) + 2m_c} \right]}.$$

Substituting (26c) in (25a) and rearranging, we can write

(28) $s_B k_B X + m_B k_B X$

$$= X - \frac{1}{n+1} m_A k_A X - \frac{n}{n+1} m_C X \frac{m_A k_A - m_B k_B}{s_C(n+1) + 2m_C}.$$

Following the same procedure to which (27) was subjected, we obtain

$$(28a) \quad k_B = \frac{1 - m_A k_A \left[\dfrac{1}{n+1} + \dfrac{n}{n+1} \dfrac{m_C}{s_C(n+1) + 2m_C} \right]}{s_B + m_B \left[1 - \dfrac{n}{n+1} \dfrac{m_C}{s_C(n+1) + 2m_C} \right]}.$$

For a term which occurs repeatedly in equations (27c) and (28a) we introduce a short notation

$$(29) \qquad \gamma = \frac{n}{n+1} \frac{m_C}{s_C(n+1) + 2m_C}.$$

This term contains n and the marginal propensities of C.

Substituting (29), we get for (27c) the streamlined

$$(30) \qquad k_A = \frac{1 - m_B k_B \left(\dfrac{1}{n+1} + \gamma \right)}{s_A + m_A(1 - \gamma)},$$

and for (28a) the analogous

$$(31) \qquad k_B = \frac{1 - m_A k_A \left(\dfrac{1}{n+1} + \gamma \right)}{s_B + m_B(1 - \gamma)}.$$

Now substituting (31) in (30), we obtain

$$(32) \quad k_A = \frac{1}{s_A + m_A(1 - \gamma)}$$
$$- \frac{m_B \left(\dfrac{1}{n+1} + \gamma \right)}{s_A + m_A(1 - \gamma)} \frac{1 - m_A k_A \left(\dfrac{1}{n+1} + \gamma \right)}{s_B + m_B(1 - \gamma)}.$$

Multiplying out, this becomes

$$
(32a) \quad k_A = \frac{1}{s_A + m_A(1 - \gamma)}
$$

$$
- \frac{m_B \left(\dfrac{1}{n + 1} + \gamma \right)}{[s_A + m_A(1 - \gamma)][s_B + m_B(1 - \gamma)]}
$$

$$
+ \frac{m_A m_B k_A \left(\dfrac{1}{n + 1} + \gamma \right)^2}{[s_A + m_A(1 - \gamma)][s_B + m_B(1 - \gamma)]}
$$

and, transporting the last term over to the left in order to lift out the k_A which it contains,

$$
(32b) \quad k_A \left\{ 1 - \frac{m_A m_B \left(\dfrac{1}{n + 1} + \gamma \right)^2}{[s_A + m_A(1 - \gamma)][s_B + m_B(1 - \gamma)]} \right\}
$$

$$
= \frac{1}{s_A + m_A(1 - \gamma)} - \frac{m_B \left(\dfrac{1}{n + 1} + \gamma \right)}{[s_A + m_A(1 - \gamma)][s_B + m_B(1 - \gamma)]}.
$$

Transporting the brace-bracketed term back to the right (which turns it upside down) and disposing of the 1 which it contains, we get

$$
(32c) \quad k_A =
$$

$$
\frac{\left\{ \dfrac{1}{s_A + m_A(1 - \gamma)} - \dfrac{m_B \left(\dfrac{1}{n + 1} + \gamma \right)}{[s_A + m_A(1 - \gamma)][s_B + m_B(1 - \gamma)]} \right\}}{\left\{ \dfrac{[s_A + m_A(1 - \gamma)][s_B + m_B(1 - \gamma)]}{[s_A + m_A(1 - \gamma)][s_B + m_B(1 - \gamma)] - m_A m_B \left(\dfrac{1}{n + 1} + \gamma \right)^2} \right\}}.
$$

Multiplying out, this becomes after convenient cancellations

(32d) $k_A =$

$$\frac{s_B + m_B(1 - \gamma) - m_B\left(\dfrac{1}{n+1} + \gamma\right)}{[s_A + m_A(1 - \gamma)][s_B + m_B(1 - \gamma)] - m_A m_B\left(\dfrac{1}{n+1} + \gamma\right)^2}$$

$$= \frac{s_B + m_B\left(\dfrac{n}{n+1} - 2\gamma\right)}{[s_A + m_A(1 - \gamma)][s_B + m_B(1 - \gamma)] - m_A m_B\left(\dfrac{1}{n+1} + \gamma\right)^2}.$$

An analogous formula can be derived for k_B and is reproduced in the text (p. 113).

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